Dynamic Vibration Absorbers

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The design and application of dynamic vibration absorbers to control machinery vibration problems is reviewed. Three case histories are presented to demonstrate the techniques.

Dynamic vibration absorbers (DVAs) can be a highly effective way to address resonant frequency problems. However, they should not be used to mask the effects of machine faults and they are good for only one frequency. In order to be effective they must be designed with the proper mass ratio and the spring must be designed to have sufficient strength to absorb the energy of the primary system. For addressing resonant frequency problems in existing equipment, a DVA is often the simplest and most cost effective solution.

Single Degree of Freedom Systems

For vibration control purposes, most systems can be modeled as having a single degree of freedom (DOF). Figure 1 shows a model of such a system. In such a system, the undamped natural frequency can be determined with the following equation:

$$\omega = \sqrt{\frac{K}{M}} \tag{1}$$

where ω is the system natural frequency in radians/sec and K and M are the effective spring constant and mass, respectively. The natural frequency units can be changed to CPM (Cycles Per Minute) with the following equation:

$$f_n = \frac{60}{2\pi}\omega \tag{2}$$

The next question is, "What will be the amplitude of vibration?" In resonant systems, it is often more convenient and informative to determine the amplification factor than the actual vibration amplitude. The amplification factor is a comparison between the static and dynamic displacement. The static displacement is simply the amount the system moves when a given force is applied to it.

$$X_0 = \frac{P_0}{K} \tag{3}$$

Figure 2 shows amplification factor versus resonance frequency ratio for systems with several different critical damping ratios. The amplification factor is the ratio of the dynamic to the static displacement, and is given by the following equation:

$$AF = \frac{X}{X_0} = \sqrt{\frac{K^2 + (c\omega)^2}{\left(K - M\omega^2\right)^2 + (c\omega)^2}} \tag{4}$$

So, if you have a resonant frequency problem, what can you do about it? There is a variety of approaches. The system can be stiffened, or mass can be added. However, the resonance frequency is a function of the square root of either of these parameters, so a significant change in either mass or stiffness may have to be added in order to effect a significant change in the resonance frequency. Damping can be added. Although this can be effective, it is often difficult to add damping to existing structures. The forcing frequency can be changed. But, in existing equipment, this is often not an option. The configuration of the restraints can be modified. This is often a simple and highly effective method to control resonant frequency vibration problems. It is a subject that could and may, in the future, have an entire article dedicated to it. At this time it will not be covered. Another approach is to add a dynamic vibration absorber (DVA) – which is the subject of this article.

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Two Degrees of Freedom Systems

With the addition of a DVA, a single degree of freedom system is converted into a two degree of freedom system as shown in Figure 3. The undamped dynamic motion amplification factor of each mass M_1 and M_2, respectively, will be defined by the following equations:

$$\frac{X_1}{X_0} = \frac{1-\left(\frac{\omega_f}{\omega_2}\right)^2}{\left(1+\frac{K_2}{K_1}-\left(\frac{\omega_f}{\omega_1}\right)^2\right)\left(1-\left(\frac{\omega_f}{\omega_2}\right)^2\right)-\frac{K_2}{K_1}} \tag{5}$$

$$\frac{X_2}{X_0} = \frac{1}{\left(1+\frac{K_2}{K_1}-\left(\frac{\omega_f}{\omega_1}\right)^2\right)\left(1-\left(\frac{\omega_f}{\omega_2}\right)^2\right)-\frac{K_2}{K_1}} \tag{6}$$

where:
X_1/X_0 = the amplification factor of the primary system
X_2/X_0 = the amplification factor of the secondary system
ω_f = the forcing frequency
ω_1 = the resonant frequency of the primary system
ω_2 = the resonant frequency of the secondary system
K_1, K_2 = the spring constants of the primary and secondary systems, respectively.

Now, this is an awesome looking pair of equations and they will define the undamped amplification factor for any two DOF system. Something interesting occurs in the special case when the secondary system is tuned to the forcing frequency. The numerator of the first equation, which is the amplification factor of the primary system, goes to zero. Figure 4 shows the undamped amplification factor of the primary system after a DVA has been attached. Note that there are now two resonance frequencies which is characteristic of a two DOF system and that the amplification factor at the original resonance frequency is now zero. This will occur when the following equation is satisfied:

$$\omega_f = \sqrt{\frac{K_2}{M_2}} \tag{7}$$

This is the principle of the DVA.

There are some precautions and limitations of a DVA. For one thing, a DVA is only good for one frequency. Also, I recommend that DVAs not be used to mask potentially dangerous machine faults. For example, a properly tuned DVA can mask the effects of unbalance. The unbalance, however, would still be beating the bearings and now the bearing supports are much more rigid which would further increase the load on the bearings. This could lead to decreased bearing life. Still, in spite of their limitations for controlling the vibration amplitude of an existing piece of equipment that has a resonant frequency problem, a DVA is often a simple and cost effective solution.

An important parameter in designing a DVA is the mass ratio. The mass ratio is the ratio of the mass of the secondary system to that of the primary system. The larger the mass ratio, the farther the two resonance frequency peaks, as seen in Figure 4, will be apart. In Figure 4, the mass ratio is 20%. It is possible to make a graph of the two frequencies at which the resonances will occur as a function of mass ratio. Figure 5 is such a graph.

From Figure 4, it is apparent that DVAs are most effective for equipment that has a relatively small range of operating

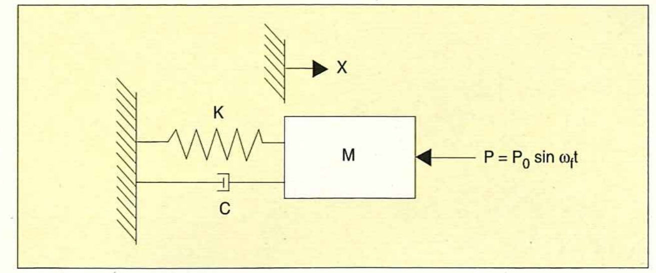

Figure 1. Single degree of freedom model consisting of one effective spring element K, one mass element M and a damping element C. The forcing function P is a sinusoidal function with peak amplitude P_0. The displacement of the system X is measured from the neutral position.

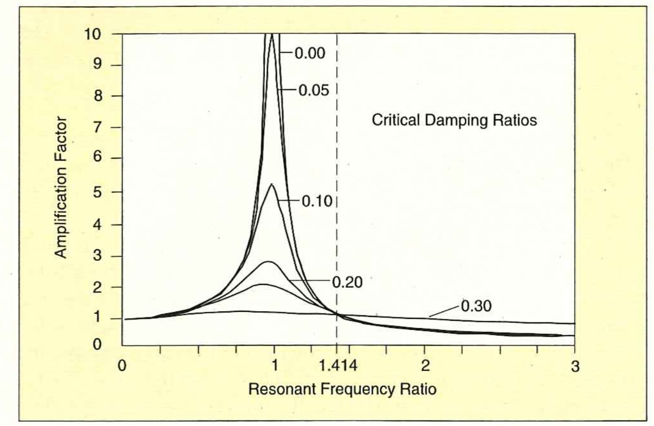

Figure 2. Amplification factor for selected critical damping ratios.

speeds. For example, a 1% mass ratio will move the resonance frequency peaks approximately 5% up and down. If this were a single speed, 3600 RPM motor that had a nuisance resonance at running speed, the resonant frequency would be moved approximately 180 RPM. In most cases, this is more than adequate to address the resonance. On the other hand, if it is necessary to move the resonance frequency more than about 30%, the mass of the secondary system becomes too large to be practical.

Motor Vibration

Now, let's consider the following problem. A motor has a range of operating speeds of 3520-3560 RPM. It weighs 400 lbs and has a resonance frequency at running speed (3540 RPM) with an amplitude of approximately 1 in/sec. How do we design a DVA to address this situation?

First, the spring configuration will be selected. There are a number of configurations that can be used, but the cantilevered beam or 'tuning fork' arrangement as shown in Figure 6 is more often than not the best system.

The next item is the mass ratio. Because the range of operating speeds is so narrow, a 1% mass ratio will be adequate. Thus, the required effective mass of the DVA is only 4 lbs. The next item is the spring. The two most important factors to be considered when designing the spring are the reaction force and the method of attachment. Force is determined from the mass and vibration of the primary system. The DVA must absorb the energy of the primary system. This energy can be determined by the classic equation:

$$F = Ma \tag{8}$$

where:
F = the force generated by the primary system which the DVA must absorb
M = the effective mass of the primary system
a = the acceleration of the primary system.

In an ideal system, there is a massless spring with a concentrated inelastic mass at the end. However, real systems are generally continuous with no definite point where the spring stops and the mass begins. In spite of this limitation, it is possible to derive a useful approximation for the effective mass. In most free standing systems, the effective mass is approximately 90% of the total mass. Thus, the effective mass M_{eff} would be

$$M_{eff} = 90\%\ M_{total} \tag{9}$$

$$M = \text{Weight / Gravity} \tag{10}$$

$$M_{eff} = 0.9 \times 400\ \text{lb} / 386\ \text{in/sec}^2 = 0.93\ \text{in-lb/sec}^2$$

The acceleration can be determined from the vibration. The average acceleration is:

$$a_{avg} = dV/dt \tag{11}$$

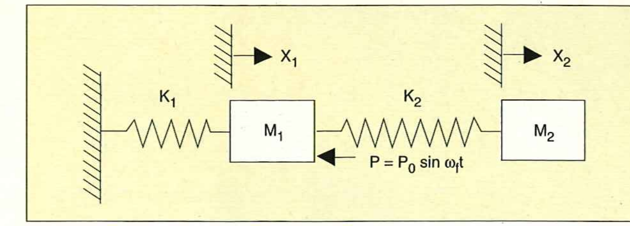

Figure 3. A model of a two degree of freedom system.

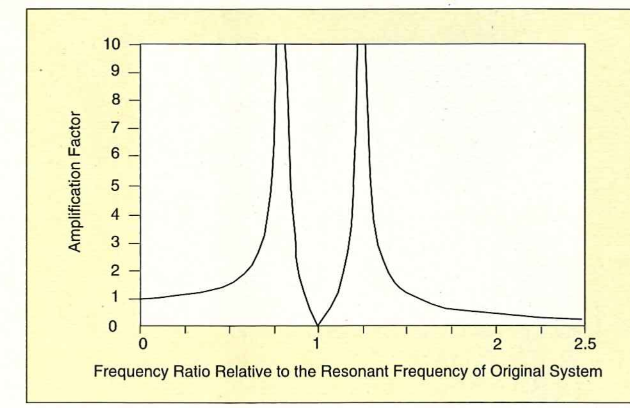

Figure 4. Amplification factor versus frequency ratio for a system with a DVA attached mass ratio of 20%.

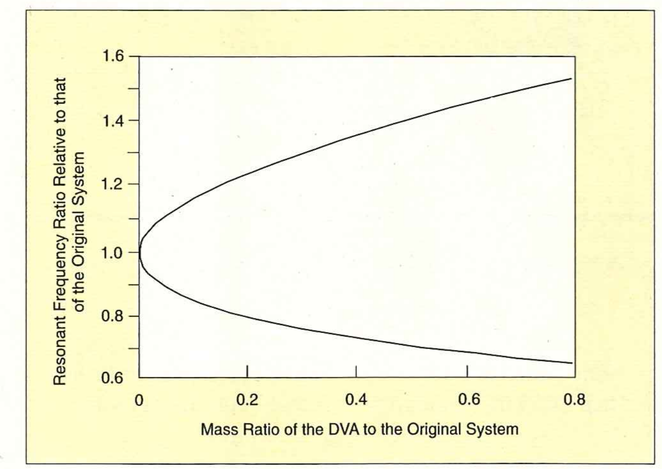

Figure 5. DVA resonant frequency ratio as a function of mass ratio.

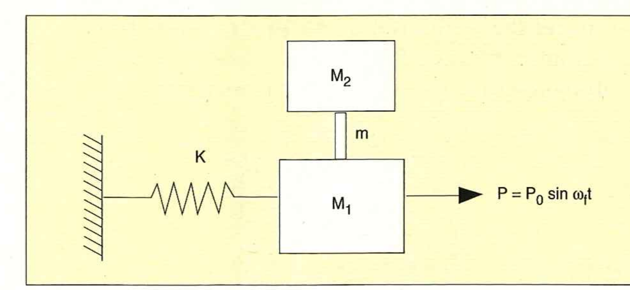

Figure 6. Schematic representation of a primary system with a cantilevered DVA attached.

where:
a_{avg} = the average acceleration
dV = the change in velocity, 1 in/sec
dt = the elapsed time.

The elapsed time is the time it takes the velocity to go from 1 to 0 in/sec which is 1/4 of a cycle. Thus:

$$dt = \left(\frac{1}{3540R/\min}\right)\left(60\frac{\sec}{\min}\right)\left(\frac{1R}{4}\right) = 0.0042 \text{ sec}$$

and thus,

$$a_{ave} = 1 \text{ in/sec} / 0.0042 \text{ sec} = 238 \text{ in/sec}^2$$

The spring has to be designed for the maximum energy not the average energy. Assuming the vibration is sinusoidal:

$$a_{peak} = 1.57 a_{ave} = 374 \text{ in/sec}^2 \qquad (12)$$

and,

$$F = 0.93 \text{ in-lb/sec}^2 \times 374 \text{ in/sec}^2 = 348 \text{ lb}$$

The next step is to select a design for the spring. Often, it is better to use more than one spring. This spreads out the load, and applies the reaction force more symmetrically about the structure. In this example, two threaded rods 2 in. diameter and 30 in. long will be employed. Each spring weighs approximately 23.4 lb (46.8 lb total). The effective mass (or weight) of various types of springs can be found in reference sources. For a cantilevered beam, the effective mass is 0.23 times the total. Thus, the effective mass of both springs before any weights are attached is approximately 10.8 lb. This is well above the 1% target mass ratio, so the effective frequency range of the DVA will be wide enough to accommodate the operating range of the motor.

The next step is to determine the stress in the spring. This can be done with the following equation:

$$\sigma = S\frac{Mc}{I} \qquad (13)$$

where:
σ = the maximum stress in the spring
S = a stress riser due to the threads, approximately 4
M = the moment being imparted on the spring
c = the distance from the neutral axis to the most extreme fibers in the cross section
I = the area moment of inertia of the cross section.

The design stress is the fatigue limit of the spring material. This is the stress level to which the spring material can be subjected for an infinite number of cycles and not fail. For mild steel, this is approximately 30,000 lb/in^2. The moment M is the product of the force times the effective length of the spring. The effective length of the spring is somewhat subjective because at this point, we do not know exactly where the concentrated mass will be located. The effective length of a uniform, cantilevered beam is 2/3 of its total length. Let's assume that the concentrated weight will be located at the same position. Therefore, the moment on each spring will be:

$$M = 20 \text{ in} \times 348 \text{ lb} / 2 \text{ springs} = 3480 \text{ in-lb / spring}$$

In determining the next two parameters c and I, the diameter of the section must be known. A threaded rod has an outside diameter and a root diameter. Which does one use? Of the two, the root diameter is better. However, the best diameter is the diameter of the stress area. This is slightly larger than the root diameter. For a 2 in. coarse thread, the diameter of the stress area is 1.78 in. The c parameter is simply the radius of the stress area or 0.89 in. For a round cross section, the area moment of inertia can be determined by the following equation:

$$I = \frac{\pi}{4}\text{Radius}^4 = \frac{\pi}{4}(0.89 \text{ in})^4 = 0.493 \text{ in}^4 \qquad (14)$$

Thus,

$$\sigma = 4\,\frac{3450 \text{ in-lb} \times 0.89 \text{ in}}{0.493 \text{ in}^4} = 24{,}900 \text{ lb/in}^2$$

Figure 7. Emergency DC lube oil pump motor with two DVAs attached.

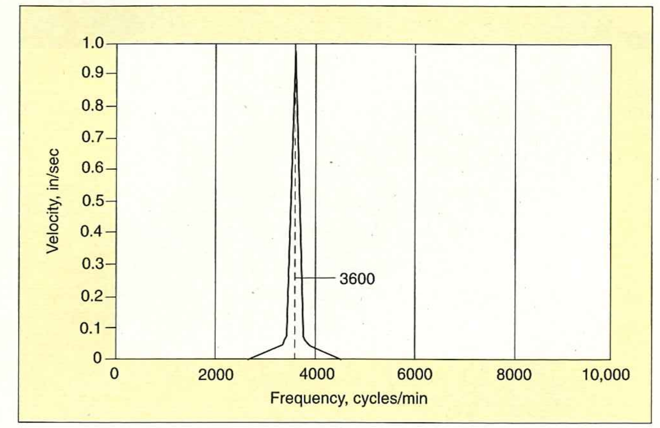

Figure 8. Frequency spectrum of a pump before DVA installation.

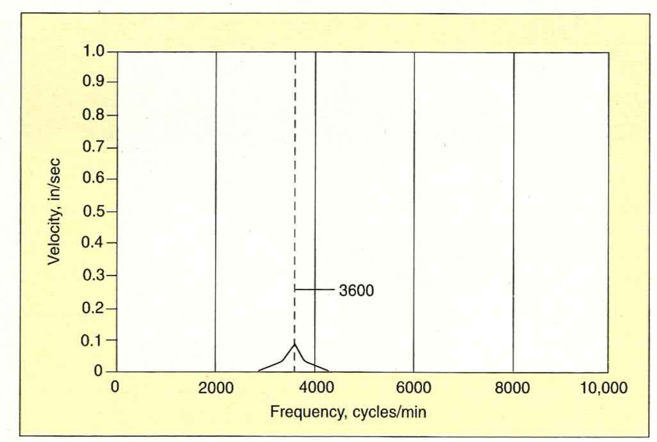

Figure 9. Frequency spectrum of the same pump operating under similar conditions after DVA installation.

This stress value is within the acceptable limit.

The next step is to tune the DVA. As mentioned earlier, the DVA will be tuned such that, from Equation (7):

$$\omega_f = \sqrt{\frac{K_2}{M_2}}$$

The frequency of the vibration to be absorbed is:

$$\omega_f = \left(3540\frac{R}{\min}\right)\left(\frac{1\min}{60\sec}\right)\left(2\pi\frac{\text{rad}}{R}\right) = 371 \text{ radians/sec}$$

The spring constant of a cantilevered beam is:

$$K = \frac{3EI}{L^3} \qquad (15)$$

where:
E = the elastic modulus of the spring material, 30×10^6 PSI for steel
L = the effective length of the spring, 20 in
$K = 3 \times 30 \times 10^6$ lb/in^2 × 0.493 in^4 / (20 in)^3 = 5540 lb/in.

Now, the forcing frequency and spring constant are known.

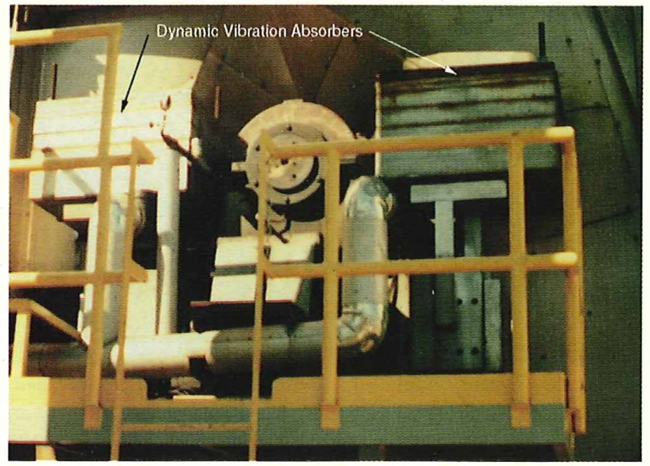

Figure 10. One of two variable speed, 5000 horsepower induced draft fans that are part of a 400 MW power plant. Each fan was found to have a resonance at full speed. As a result, unit output had to be reduced. The problem was solved by installing four DVAs on each fan. Each has an effective mass of over one ton.

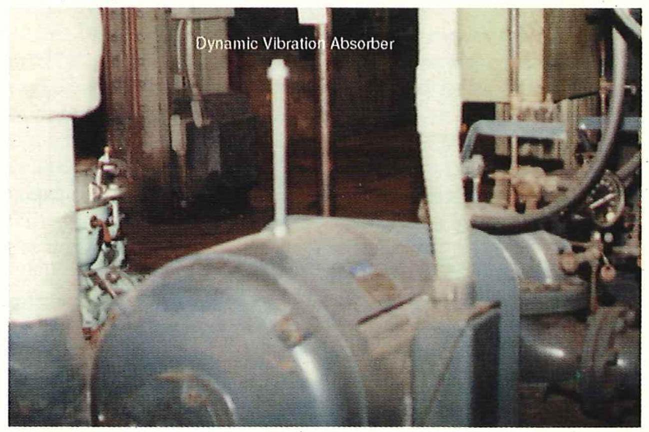

Figure 11. A 75 horsepower motor. The DVA is simply a threaded rod inserted into the lifting hole. The motor has a resonance at 2x running speed. Before the DVA was installed, the vibration amplitude was approximately 0.40 in/sec. After it was installed, the level of vibration was approximately 0.14 in/sec. Note that the rod and nut are blurry which indicates that they are vibrating.

By rearranging terms in the resonance frequency equation, the effective mass can be determined:

$$M_{\text{eff}} = \frac{K}{\omega^2} = 5540\ \text{lb/in} \,/\, (374\ \text{in/sec})^2 = 0.04\ \text{lb-sec}^2/\text{in} \quad (16)$$

$$\approx 15.6\ \text{lb/DVA}$$

It was mentioned earlier that the effective mass is a combination of the mass of the spring and the concentrated mass. Thus, the effective mass of the DVA can be determined with the following equation:

$$M_{\text{eff}} = M + 0.23m \quad (17)$$

where:

M = the mass of the concentrated weight
m = the mass of the spring (the threaded rod).

By rearranging terms, the size of the concentrated weight can be determined:

$$M = M_{\text{eff}} - 0.23m = 15.6\ \text{lb} - (0.23 \times 23.4\ \text{lb}) = 10.3\ \text{lb}$$

The simplest way to fabricate the concentrated weight is to sandwich a stack of washers between two nuts on the threaded rod. The DVA can be fine tuned by moving the weight up and down the threaded rod. The DVA will resonate when it is tuned and vibrate out of phase with the forcing function. There is, however, one precaution. If the mass ratio is small, the difference between the target tuning frequency, the resonant frequency of the primary system and one of the resonant frequencies of the combined system will be small.

Figure 7 shows two DVAs installed on the pump motor. In this case the subject pump ran fine under its own power, ~ 1200 RPM. However, when the pump beside it ran, ~ 3600 RPM, it vibrated severely. The condition was successfully addressed by installing the DVAs.

The next two illustrations indicate how effective a DVA can be. Figure 8 shows the frequency spectrum of the pump in Figure 7 with the 3600 RPM pumps running before the DVAs were installed. The amplitude of vibration at the forcing frequency is just under 1 in/sec. Figure 9 shows the same data after the DVAs were installed. In this frequency spectrum, the amplitude of vibration is less than 0.1 in/sec.

Large and Small DVAs

DVAs come in a variety of sizes. Figure 10 shows a very large DVA system. It is installed on one of two induced draft fans. These fans are part of a 400 MW electric generating plant. Fan speed is variable with higher speeds occurring at higher loads. During unit start up testing, a resonance condition was found in the fans at full load operating speed. As a result, full power output had to be reduced by approximately 1/3. The condition was successfully addressed by installing the DVA system.

Figure 11 shows a small DVA installed on a motor. Initially, this motor was found to have a very high level of vibration, approximately 2 in/sec, at 2x running speed. As a result, the motor was a chronic, high maintenance item. Balancing the rotor had little affect. A resonance condition was subsequently confirmed. Most of the vibration was eliminated by installing resilient mountings on one side of the motor. This is an example of a restraint configuration modification mentioned in the first part of this article. This reduced the level of vibration from approximately 2.0 to 0.4 in/sec. Although this was a dramatic drop in vibration, it was desirable to reduce the level of vibration below 0.15 in/sec. A very simple DVA was designed and installed. This reduced the vibration to 0.14 in/sec. S/V