Hoop frequency as a predictor of performance for softball bats

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ABSTRACT: Hollow metal and composite baseball and softball bats exhibit two types of vibrational modes, bending modes and hoop modes. The hoop modes are unique to hollow bats and involve only a radial vibration of the barrel of the bat. The lowest frequency hoop mode is responsible for the both the "ping" sound of a metal bat and the so-called "trampoline effect." Modal analysis is used to determine the mode shapes and frequencies for a wide variety of softball bats. Hoop mode frequencies are shown to separate families of bats (*ie.*, single-wall, multi-wall, and composite) according to barrel construction. A simple mass-spring model of the trampoline effect suggests that hoop frequency might be correlated to measured performance. A general trend is observed that higher performance bats, with respect to batted ball speed, tend to have lower hoop frequencies. This result suggests that hoop frequency might be one explanation for differences in performance between different types of hollow softball bats.

INTRODUCTION

Since the introduction of hollow metal bats in the early 1970's advances in design aided by improvements in materials and construction methods have led to a steady increase in the performance of baseball bats and especially softball bats. Concerns over safety and a desire for balance between offense and defense have led league officials and governing bodies to either ban certain bat models or place limits on bat performance. ASTM standards have been developed to measure performance and are being applied to determine which bats are legal for play.

There have been relatively few experimental studies comparing the performance of various types of hollow bats. A batting cage study (Greenwald *et. al.*, 2001) showed that metal bats outperformed wood bats. A follow up study (Crisco *et. al.*, 2002) attributed the difference between metal and wood bats to an increased swing speed, due the metal bats having a lower moment of inertia, and an inherent elastic property of the metal bats. The existence of a "trampoline effect" is given as a reason for higher performance, but no indication is given as to why some metal bats perform better than others. Several other studies of bat performance (Fallon *et. al.*, 2000; Sherwood *et. al.*, 2000; Nathan, 2003) acknowledge the fact that hollow bats exhibit a hoop mode, yet none appear to have used the hoop mode frequency either as a criteria for analyzing their data or as a validation of computational models. Certainly there has been no study comparing the hoop mode frequencies of hollow bats in relation to measured performance. This paper attempts to identify hoop frequency as a parameter which might help explain differences in performance which exist between various types of hollow bats.

VIBRATIONAL MODES OF A HOLLOW BAT

The vibrational mode shapes and frequencies of a baseball or softball bat are easily determined by a modal analysis experiment. All baseball and softball bats exhibit bending, or flexural, modes which involve the entire length of the bat. The first four bending modes of a high performance composite softball bat are shown on the left side of Fig. 1. Nodes are indicated by the dots; the first and second bending modes have nodes located at 7 and 5 inches from the barrel end, respectively. The region between these two nodes is referred to as the "sweet zone" (Cross, 1998) since impacts within this region will only poorly excite the first two bending modes, thus reducing the amount of initial ball kinetic energy lost to bat vibrations.

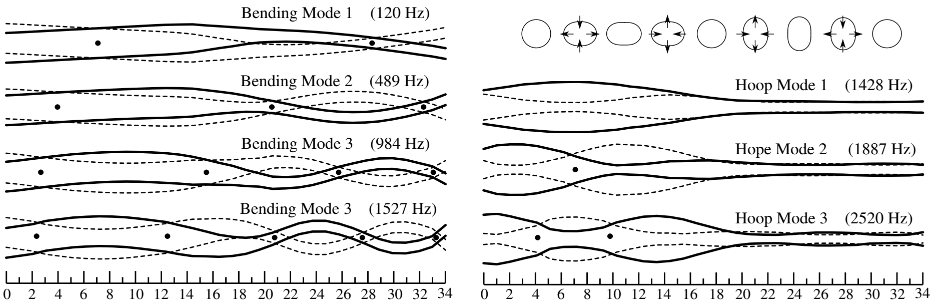

Figure 1. Mode shapes and frequencies for the first four bending modes (left) and the first three hoop modes (right) of a high performance composite softball bat. Nodes are indicated by the dots. Scale is in inches. Handle is at the right.

The right side of Fig. 1 shows length profiles of the first three hoop modes, unique to hollow bats. These mode shapes involve a radial oscillation of the barrel only. The inset at the top of the figure shows one cycle of the cross-section of the oscillation of the bat barrel for a each of the first three hoop modes. The fundamental hoop mode is responsible for the ping sound of a metal bat. The first hoop mode is also responsible for the trampoline effect. The barrel of the bat essentially acts as a spring, compressing when a ball impacts the bat. The more the bat compresses, the less energy is lost in the reduced compression of the ball, and the ball rebounds from the bat with greater speed than it would have from a solid wood bat.

In well designed bats the antinode of the first hoop mode lines up with the node of the first bending mode, so that an impact at the sweet spot will not lose energy to the bending modes but will result in maximum deflection of the barrel, thus increasing the energy returned to the ball after the collision and resulting in a higher batted ball speed. The shape of the first hoop is fairly flat over a region 4 to 10 inches from the barrel end. This may be responsible for the wider sweet spot observed for metal bats, if the increase in performance due to the trampoline effect in the hoop mode is greater than the performance loss caused by excitation of the first two bending modes for an impact away from the sweet spot.

SIMPLE MASS-SPRING MODEL OF THE TRAMPOLINE EFFECT

A simple model which explains the trampoline effect for a hollow bat is a modified nonlinear mass-spring model which has been successfully used to describe the collision between golf ball and club (Cochran, 2002). As shown in Fig. 2, the ball is modeled as a nonlinear mass-spring system with an initial downward velocity and the bat is modeled as a linear mass-spring system mounted on a fixed base. A nonlinear model is needed for the ball to adequately account for the hysteresis which occurs during compression and relaxation of the ball. During the collision the bat and ball behave as a coupled system, and the collision is taken to be over when the ball mass reaches its maximum velocity after beginning to rebound upwards.

The differential equations describing this mass-spring model are:

$$m_1\ddot{x}_1 = -s_1(x_1 - x_2)|x_1 - x_2|^a + c_1(\dot{x}_1 - \dot{x}_2)|x_1 - x_2|^b \tag{1a}$$

$$m_2\ddot{x}_2 = -s_2x_2 - c_2\dot{x}_2 - s_1(x_1 - x_2)|x_1 - x_2|^a + c_1(\dot{x}_1 - \dot{x}_2)|x_1 - x_2|^b \tag{1b}$$

where the nonlinear behavior of the ball is characterized by the two terms with exponents a and b. For our model the following parameters were assumed as an approximation of the behavior of a 375/.44 softball: c_1=4700, s_1=40.6E6 N/m, m_1=0.180 kg, c_2= 100, m_2=0.16 kg, with a=0.65, and b=0.5. The spring constant for the bat was determined by the the hoop mode frequency and m_2. The plot in Fig. 2 was generated by numerically solving Eqs. (1a) and (1b) for the maximum upwards velocity of the ball for each hoop frequency value. The results have been expressed in terms of a collision efficiency which is simply the ratio of final to initial ball speeds. The plot has been normalized to the value for a rigid bat, so the curve represents the improvement in performance one might expect to see for a bat with a given hoop frequency compared to a solid bat which does not have a hoop mode.

The shape of the curve in Fig. 2 indicates that as the hoop frequency decreases the resulting batted-ball speed increases, the effect becoming most dramatic as the frequency range decreases from 2000 Hz to 1000 Hz. Bats with hoop frequencies within this range should exhibit a considerably higher performance compared to wood

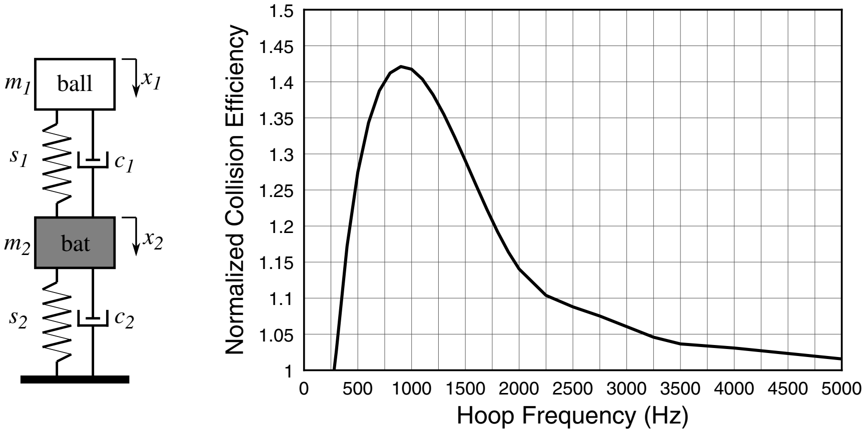

Figure 2. (left) Mass-spring model of the trampoline effect. (right) Results from simple mass-spring model showing how the normalized ratio of outgoing to incoming ball speed depends on the hoop frequency of the bat.

bats, with performance increasing as hoop frequency decreases. If the hoop frequency drops below 900 Hz the bat barrel spring becomes too soft and the collision quickly becomes very inefficient.

HOOP FREQUENCY AND BARREL CONSTRUCTION

An important question is whether or not the simple model in Fig. 2 accurately predicts the performance of real bats. Research has shown that lowering the first hoop frequency and raising the first bending frequency increases the ball-bat coefficient of restitution (Naruo & Sato, 1997). It would be instructive, then, to examine the bending and hoop frequencies of different types of hollow bats. Figure 3 shows a plot of first bending frequency versus first hoop frequency for 56 slow pitch softball bats covering a wide variety of performance and construction. For the vast majority of softball bats tested the first bending frequency falls between 100 and 200 Hz. The two exceptions with bending frequencies near 275 Hz are "bottle bats" with 20-inch barrels; the longer barrel increases the bending stiffness.

The hoop mode frequencies show a much greater spread than the bending modes, covering the entire range from 1000 Hz to 2500 Hz. This wide spread of hoop mode frequencies is much more pronounced for slow pitch softball bats than it is for youth and adult baseball bats. Most commercially available metal baseball bats have hoop frequencies between 1900-2000 Hz. In 1989 graphite baseball and softball bats were introduced and marketed as having the strength of aluminum with the performance of wood. These early graphite bats have hoop frequencies above 3300 Hz and according to the simple model in Fig. 2, the trampoline effect would be almost negligible. Because of their low performance graphite bats did not remain on the market for long. Incidentally, a corked wood baseball bat has been found to have a hoop frequency around 5500 Hz which is much too high for a noticeable trampoline effect to exist.

Looking with closer detail at the spread of hoop frequencies in Fig. 3, it becomes apparent that different types of barrel construction tend to fall within specific frequency ranges. Single-walled bats, whose barrels are constructed from a single

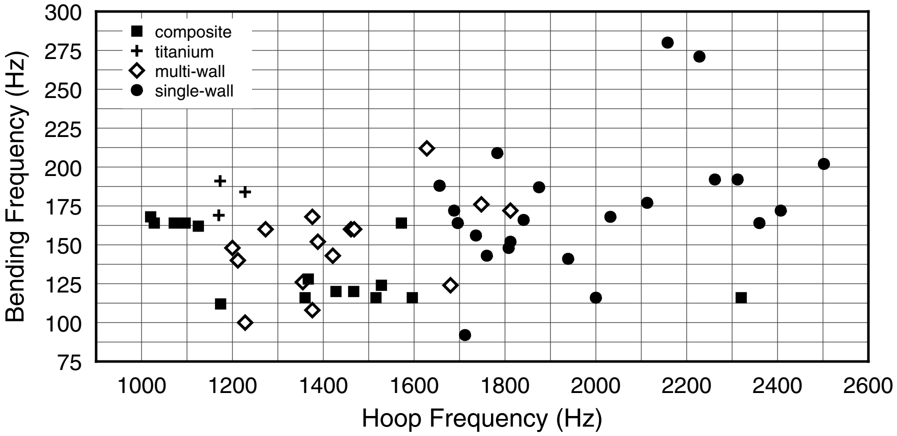

Figure 3. Softball bats may be separated according to type of barrel construction by plotting the frequency of the first bendng mode versus the frequency of the first hoop mode.

layer of aluminum, all have hoop frequencies above 1650 Hz. Even with the advanced alloys available today it is practically impossible to make an aluminum single-walled bat with a hoop frequency below 1600 Hz that is strong enough to not dent when an average player makes solid contact with a ball.

Single-walled titanium softball bats were introduced in 1993. The high strength of titanium allowed bat barrels to be made much thinner than is possible with aluminum and yet still withstand the force of a collision with a ball without denting. As shown in Fig. 3 these titanium bats have hoop frequencies around 1200 Hz. At the time titanium bats were introduced, most single-walled aluminum bats had hoop frequencies up around 2000 Hz. The huge difference in hoop frequency could explain the huge performance advantage which quickly resulted in a ban on titanium bats.

The desire to produce higher performing bats using primarily aluminum alloys led designers to begin experimenting with multiple walled bats, which have barrels consisting of two or more layers of metal or metal/composite hybrids. The advantage of a multi-wall bat is that each individual wall is thinner, lowering the effective spring constant, while the strength comes from the sum of the layers. Thus, multi-wall bats tend to have lower hoop frequenices than single-walled bats. As shown in Fig. 3, hoop frequencies for most multi-wall bats tested fall between 1200 and 1500 Hz.

Composite softball bats made a significant entrance to the market in 2000, with recent high performance composite bats producing nearly the same reaction from league officials and governing bodies as titanium bats did. The highest performing softball bats currently available are all-composite bats. Composite materials have the advantage of being anisotropic, allowing the bending and hoop stiffnesses to be modified relatively independently of each other. This means that a composite bat can have almost any hoop frequency regardless of its bending frequency. Figure 3 shows composite bats with hoop frequencies as low as 1000 Hz and as high as 2300 Hz. There is a group of high performance composite bats which perform as well as some of the best multi-wall bats on the market. As shown in Fig. 3, this group of composite bats have hoop frequencies in the range of 1350-1600 Hz, overlapping the range of hoop frequenices for multi-wall bats. There is also a group of very high performing composite bats which are currently the highest performing softball bats available. According to Fig. 3 their hoop frequencies fall between 1000 and 1150 Hz, having the lowest range of hoop frequenices of any commercially available softball bats to date.

HOOP FREQUENCY AND BAT PERFORMANCE

The simple model in Fig. 2 suggests that bats with lower hoop frequencies should produce higher batted ball speeds. This prediction would seem to be borne out by the general trends observed between families of bats shown in Fig. 3. However, a correlation between hoop frequency and a recognized bat performance metric is desirable to validate the role of hoop frequency. There are several bat performance metrics (Smith, 2001; Nathan, 2003) to which one could attempt a correlation with hoop frequency. Two metrics which one might expect to correlate well with hoop frequency are the Bat Performance Factor (BPF) and the related Bat Ball Coefficient of Restitution (BBCOR), since these two metrics depend primarily on the relative elastic constants of the bat and ball. Unfortunately, performance data for these metrics is not yet available for the bats shown in Fig. 3. What is available is some data for the Batted Ball Speed (BBS) metric as per ASTM F2219. For this metric, a softball is fired from a cannon at 110 mph towards a stationary bat. The bat is clamped at 6 inches from the knob in a mount which is free to rotate after ball impact. The ratio of incoming and outgoing ball speeds is measured using light curtains. The position on the barrel producing the maximum rebounding ball speed is located and the ratio of ball speeds is used, along with the bat's moment of inertia and the impact location, to calculate a Batted Ball Speed representative of field conditions.

Figure 4 shows measured batted ball speed versus hoop frequency for a collection of bats that were tested in a high speed impact experiment in accordance with ATSM F2219. Not all of the bats in Fig. 3 are included in Fig. 4; high speed BBS data was only available for a subset of the bats. In addition, a few bats within this subset were tested at two different laboratory facilities, which explains why some data points have two BBS values for the same hoop frequency. The curved line in the plot represents the best fit to the data. The general trend is clearly apparent, that bats with a higher batted ball speed tend to have a lower hoop frequency. The scatter in the data about this best fit curve is likely due to differences in the moment of inertia and impact location between the various bats, and also to variations in the balls used to test the bats. Batted Ball Speed depends strongly on MOI and impact location as well as the relative spring constant between bat and ball.

It is interesting to note that the data in Fig.4 shows the same groupings according to barrel construction as Fig. 3, further strengthening the idea that hoop frequency is related to performance. The single-walled bats, with hoop frequencies between 1650 and 2400 Hz all have batted ball speeds between 90 and 96 mph. The multi-walled bats, with hoop frequencies between 1200 and 1700 Hz all have batted ball speeds between 96 and 100 mph. The two titanium bats with hoop frequencies near 1200 Hz have BBS values between 100 and 103 mph. The high performance composite bats,

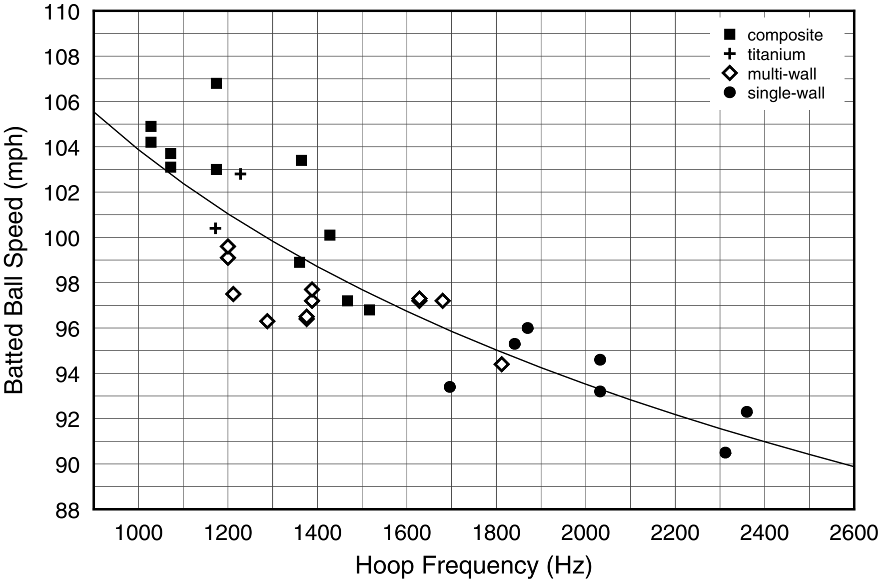

Figure 4. Measured Batted Ball Speed versus Hoop Frequency for a variety of adult slowpitch softball bats.

with frequencies between 1300 and 1600 Hz have BBS values between 96 and 100 mph, similar to the range of the mult-wall bats; one exception is a composite bat with a hoop frequency of 1360 Hz and BBS of 103.4 mph. And finally, the very high performance composite bats all have hoop frequencies below 1200 Hz and BBS results above 103 mph.

The data suggests rather strongly that hoop frequency may be an indicator of bat performance, and may explain differences in performance between different types of hollow bats. While other bat parameters, such as moment of inertia and impact location also affect performance, hoop frequency seems to correlate with performance both in terms of the historical development of hollow bats, and with respect to measured batted ball speed. Further attempts at correlating hoop frequency to BPF and BBCOR performance metrics are in progress. In the meantime, hoop frequency may prove a useful tool in designing bats to meet performance standards.

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