Spring Constants for Hockey Sticks

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Hockey sticks provide a real-world application of Hooke's law and the concept of an elastic spring constant. Most physics students are introduced to spring constants and mechanical oscillation via the standard mass-spring system, even though there are many other systems that exhibit simple harmonic oscillation.[1,2] In this paper we explore Hooke's Law through the static bending of a hockey stick and extract values for Young's modulus from measurements of force and displacement.

A hockey stick can bend an amazing amount (up to 30 degrees) during slap shots and wrist shots.[3,4] As the stick temporarily bends, it stores and then releases potential energy. In the hands of a player, a flexible stick can store more potential energy than a stiff stick.[5] However, while some studies suggest that more flexible sticks produce higher puck speeds,[6,7] other studies suggest that player skill has a greater influence on puck speed than does stick stiffness.[5,8]

Measurements of Static Bending

Hockey sticks are rated according to weight, shaft flex, and the amount of curvature in the blade. The flex rating is given as a number that roughly represents the amount of force in pounds required to produce a deflection of 1 inch in a three-point bending apparatus. Typical flex ratings are:[9] Youth = 40, Junior = 50, Intermediate = 60-75, Regular = 85, Stiff = 100, and Extra Stiff = 110. In a three-point bending test, a force F is applied at the midpoint between two supports separated by a distance L resulting in a static deflection δ at the midpoint of the beam,[10-12]

$$F = \frac{48EI}{L^3}\delta \quad , \tag{1}$$

where E is the Young's modulus of the material, and I is the second area moment of the cross section. The material properties and the dimensions of the stick are constant, so Eq. (1) is just Hooke's law for a mass-spring system, $F = k\,\delta$, where k represents an effective spring constant.

The three-point bending test for measuring hockey stick flex requires the use of a load-compression machine which, while often found in engineering departments at universities, is not readily available to students in introductory physics courses, especially at the high school level. However, if the boundary conditions are changed to those of a cantilever beam, clamped at one end and loaded with a static force at the other end, the relationship between force and deflection is[10–12]

$$F = \frac{3EI}{L^3}\delta \quad , \tag{2}$$

which differs from Eq. (1) only by a factor of 16. A cantilever beam static bending test is very accessible for introductory physics students.

Figure 1 shows our experimental setup for measuring the effective spring constant of a hockey stick. The butt end of the shaft is clamped to a rigid work-

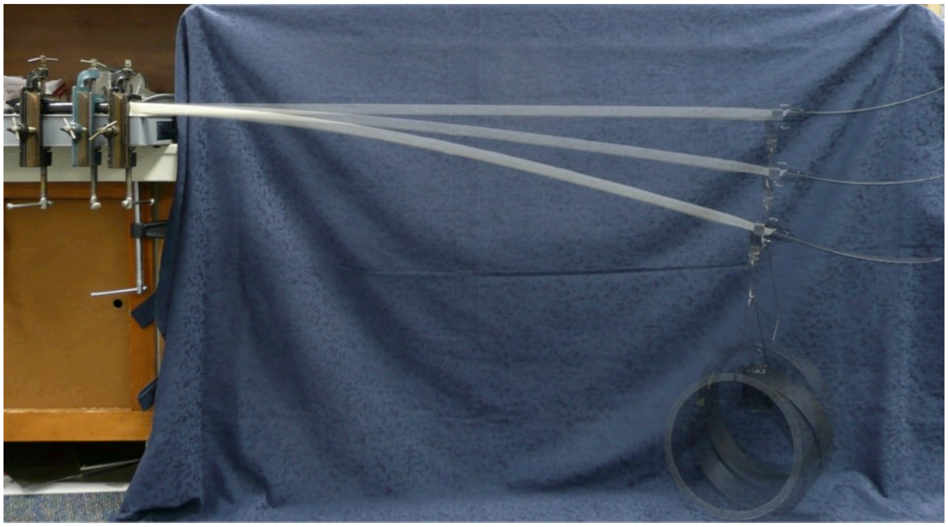

Fig. 1. Superposition of three photographs of a wood hockey stick clamped at one end and with no force, 83 N, and 130 N applied to the blade end.

clamp point, so we kept our applied forces below 100 N.

According to Eqs. (1) and (2), the effective stiffness depends on the length of the stick. Most sticks were long enough to allow a consistent length of 1.17 m between clamp and load points. For shorter sticks, we used the maximum length possible and then normalized the stiffness to the longer length.

Table 1 shows a sampling of stiffness values, as an effective spring constant in N/m, for a variety of wood, composite, and aluminum sticks with different flex ratings. Stick S1 was actually a piece of 1"×2" wood and served as a reference. Stick S3 was the composite stick that broke. Sticks S3 and S4 are commercial and professional versions of the same stick model. Sticks S6 and S18 are the same stick model in two different flex ratings. Sticks S16 and S17 are two-piece sticks with a wood blade attached to aluminum and graphite shafts, respectively.

The stiffness values in Table 1 agree very well with the range of values found in the published literature,[6,8,14,15] after normalizing the lengths. The data

bench and masses are hung from a hook at the other end of the shaft where the blade and shaft meet. We varied the amount of mass hanging from the end, and measured the resulting displacement at the load point. The slope of a force versus displacement plot represents the effective spring constant, or the shaft stiffness in units of N/m.

The cantilever value for stiffness is easily converted to the three-point test value by multiplying the cantilever slope by 16, but this conversion is really only necessary if one wants to compare stiffness values to those published in the literature. The application of a hockey stick as a real-world example of Hooke's law and static deflection does not require this conversion, and Eq. (2) will correctly differentiate between sticks of varying stiffness.

Stiffness Values for Hockey Sticks

Figure 2 shows typical force versus displacement curves for aluminum and composite hockey sticks, clamped at the handle and loaded at the blade end. The plots are surprisingly linear, even though the amount of deflection exceeds the limit over which the linear Euler-Bernoulli bending theory ought to apply.[10,13] Forces exceeding 100 N caused some of the more flexible sticks to make cracking sounds, indicating possible stress fractures and one stick broke at the

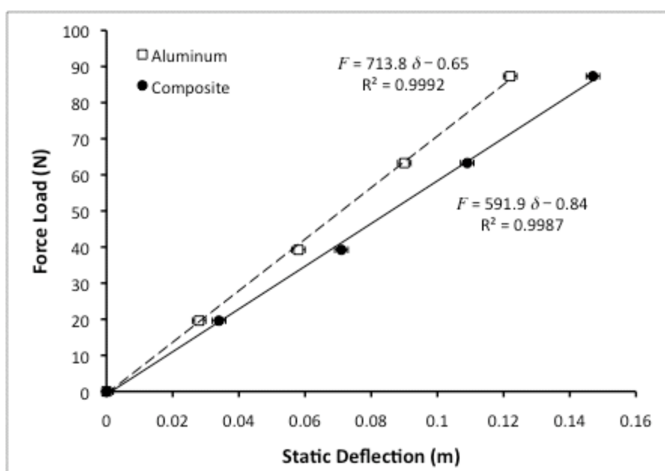

Fig. 2. Force versus displacement curves for aluminum and composite hockey sticks clamped at the butt end and loaded at the blade.

Table I. Hockey stick stiffness values (effective spring constants) for several composite, wood, and aluminum hockey sticks.

Stick	Stick Material	Flex Rating	Stiffness (N/m)	Young's Modulus (GPa)
S1	1″ × 2″ pine		3686	8.88
S2	Wood	52	7026	11.71
S3	Composite	85	6740	31.97
S4	Composite	85	8913	41.77
S5	Wood		9214	11.81
S6	Composite	85	9470	31.65
S7	Composite	85	9481	
S8	Composite	90	9237	
S9	Composite	100	8644	
S10	Composite	100	9077	
S11	Wood	85	10052	11.77
S12	Wood		10286	15.82
S13	Composite	100	10419	
S14	Composite	100	10868	
S15	Wood		10475	16.25
S16	Aluminum/wood		11420	54.62
S17	Graphite/wood	100	11939	
S18	Composite	110	11384	38.05
S19	Composite	110	13064	42.16

show that sticks with a lower flex rating do indeed have a lower effective spring constant and, like a softer spring, are easier to flex. Also, our aluminum stick is stiffer than the wood sticks while composite sticks cover the entire range of stiffness values.

Extracting Young's Modulus

Static bending tests are often used to determine the Young's modulus of wood.[12,16] Young's modulus may be obtained from the slope of a force versus displacement curve as shown in Fig. 2, using Eq. (1) or (2) with the length between clamp and load points and the second area moment of the cross-section. For a solid rectangular beam of width w and height h, the second moment of area for the cross section is[18]

$$I = \frac{w h^3}{12} \quad . \tag{3}$$

This allows us to calculate the Young's modulus for the wood sticks in Table 1. Stick S1 was made from pine, and our value of $E = 8.88$ GPa agrees favorably with the accepted value of 8.69 GPa.[17] Wood hockey sticks are usually made from either Rock Elm or Oak[2] which have Young's modulus values of 10.6 GPa and 12.3 GPa respectively.[17] Our values for wood sticks S2, S5, and S11 fall within this range. Sticks S12 and S15 have higher Young's moduli; S12 is covered in a layer of composite skin while S15 is made of 15-ply laminate, so both sticks would be expected to have higher Young's modulus values than the wood from which they are made.

The shafts of aluminum and composite hockey sticks are hollow, and they tend to be lighter weight than solid wood shafts. The second moment of area for the cross-section of a hollow rectangular tube of width w, height h and wall thickness t is[18,19]

$$I = \frac{w h^3}{12} - \frac{(w-2t)(h-2t)^3}{12} \quad . \tag{4}$$

Our measured value of Young's modulus for the aluminum stick is 54.62 GPa, which is 20% lower than the accepted value of 69 GPa. However, the aluminum shaft has rounded corners and the walls are slightly concave so our use of Eq. (4) is only an estimate. Young's moduli of several composite sticks, for which we were able to measure the wall thickness, range from about 31 to 42 GPa. This range is lower than the value for aluminum, but many of these composite sticks had thicker walls so the value of second moment of area is larger. The elastic properties of composite materials depend on the fiber material as well as the angle of the weave, and since this can vary considerably from stick to stick, and even along the length of the same stick, it is not possible to assess our values of Young's modulus for composite sticks.

Further Exploration

This experiment provides an interesting alternative for exploring Hooke's law in a manner that has relevance to a sport students might relate to. Interested students could be challenged to explore some of the following ideas.

1. Hockey stick flex ratings depend on the length of the stick and some manufacturers provide an approximate conversion for players who shorten their sticks.[9] Students could change the clamp point to

vary the value of *L*, and verify the relationship between stick stiffness and length.

2. Eq. (2) assumes that the load is applied at the end of the shaft. This equation may be modified to account for a load applied at some arbitrary location along the stick.[10,11] Students could explore static deflection as a function of load location.
3. Students could apply a load to the end of the stick and measure the deflection at intervals along length of shaft. A plot of deflection versus position along shaft could be compared to Euler-Bernoulli bending equations for static deflection of a loaded beam.

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