Absorption Coefficients and Impedance

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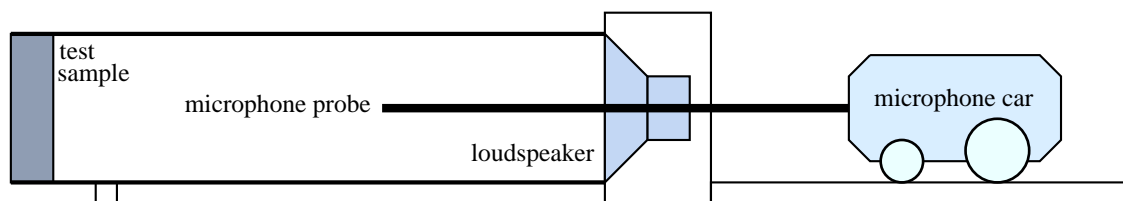
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I. Introduction and Background

In this laboratory exercise you will measure the absorption coefficients and acoustic impedance of samples of acoustic absorbing materials using the Brüel & Kjær Standing Wave Apparatus Type 4002. Absorbing materials play an important role in architectural acoustics, the design of recording studios and listening rooms, and automobile interiors (seat material is responsible for almost 50% of sound absorption inside an automobile).

The reverberation time of a room is a very important acoustic quantity of concern to both architects and musicians. The growth and decay of the reverberant sound field in a room depends on the absorbing properties of the materials which cover the surfaces of the room under analysis. The Standing Wave Apparatus Type 4002, one of Brüel & Kjær's first products (almost 50 years ago), was developed primarily to measure the sound absorbing properties of building materials, and is still widely used today. The standing wave tube (also called an impedance tube) method allows one to make quick and easy, yet perfectly reproducible, measurements of absorption coefficients. The impedance tube also allows for accurate measurement of the normally incident acoustic impedance, and requires only small samples of the absorbing material.

A sketch of the B&K Standing Wave Apparatus is shown in the figure below. A loudspeaker produces an acoustic wave which travels down the pipe and reflects from the test sample. The phase interference between the waves in the pipe which are incident upon and reflected from the test sample will result in the formation of a standing wave pattern in the pipe. If 100% of the incident wave is reflected, then the incident and reflected waves have the same amplitude; the nodes in the pipe have zero pressure and the antinodes have double the pressure. If some of the incident sound energy is absorbed by the sample, then the incident and reflected waves have different amplitudes; the nodes in the pipe no longer have zero pressure. The pressure amplitudes at nodes and antinodes are measured with a microphone probe attached to a car which slides along a graduated ruler. The ratio of the pressure maximum (antinode) to the pressure minimum (node) is called the standing wave ratio SWR. This ratio, which always has a value greater than or equal to unity, is used to determine the sample's reflection coefficient amplitude R, its absorption coefficient α, and its impedance $\mathbf{Z}(\omega)$.

An alternate method for determining the absorbing properties of a material, also in wide use today, involves placing a unit-area piece of material (say one meter squared) in a special reverberation room. The difference in the reverberation time with and without the material yields the absorbing properties of the material. This method is generally more expensive, requiring precisely calibrated sensors and a specially designed reverberation chamber, and is much less convenient. Although this method does not yield normally incident acoustic impedance data, it is superior for measuring absorption characteristics for randomly incident sound waves, and is preferable for determination of absorbing properties that depend on the size of the material.

On a historical note, water-filled impedance tubes became critically important to the development of absorptive coating for submarines during World War II. Such coatings, developed first by Germany, were used (and still are) to protect submarines from sonar detection.
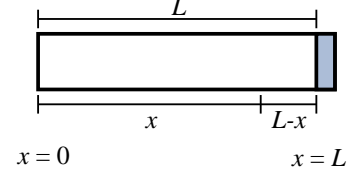
II. Theory

Assume that a pipe of cross-sectional area S and length L is driven by a piston at $x'=0$. The pipe is terminated at $x'=L$ by a mechanical impedance $\mathbf{Z}_{mL}$. If the piston vibrates harmonically at a frequency sufficiently low that only plane waves propagate[†] then the pressure wave in the pipe will be the sum of two waves travelling in the positive and negative x-directions,

$$\mathbf{p}(x,t) = \mathbf{A}e^{i\,[\omega t - kx]} + \mathbf{B}e^{i\,[\omega t + kx]} \qquad (1)$$

To simplify the math, we specify the position in the pipe in terms of $L-x$, rather than x, as shown at right. The quantity $(L-x)$ measures the distance from the termination towards the driver. With this variable change the pressure equation becomes

$$\mathbf{p}(x,t) = \mathbf{A}e^{i\,[\omega t + k(L-x)]} + \mathbf{B}e^{i\,[\omega t - k(L-x)]} \qquad (2)$$

where $\mathbf{A}$ and $\mathbf{B}$ are determined by the boundary conditions at $x=0$ and $x=L$. Using Euler's Equation $[\rho\partial\vec{u}/\partial t = -\partial p/\partial x]$, one may obtain the particle velocity in the pipe,

$$\vec{\mathbf{u}}(x,t) = \frac{1}{\rho c}\left(\mathbf{A}e^{i\,[\omega t + k(L-x)]} - \mathbf{B}e^{i\,[\omega t - k(L-x)]}\right) \qquad (3)$$

The mechanical impedance of the plane waves in the pipe may be expressed as

$$\mathbf{Z}_m(x) = \frac{\mathbf{p}S}{\vec{\mathbf{u}}} = \rho c S\left[\frac{\mathbf{A}e^{i\,k(L-x)} + \mathbf{B}e^{-i\,k(L-x)}}{\mathbf{A}e^{i\,k(L-x)} - \mathbf{B}e^{-i\,k(L-x)}}\right] \qquad (4)$$

The wave mechanical impedance at $x=L$ must equal the mechanical impedance of the termination,

$$\mathbf{Z}_{mL} = \rho c S\left[\frac{\mathbf{A}+\mathbf{B}}{\mathbf{A}-\mathbf{B}}\right] = \rho c S\left[\frac{1+\frac{\mathbf{B}}{\mathbf{A}}}{1-\frac{\mathbf{B}}{\mathbf{A}}}\right] \ . \qquad (5)$$

If we choose to write $\mathbf{A} = A$ and $\mathbf{B} = Be^{i\theta}$ then

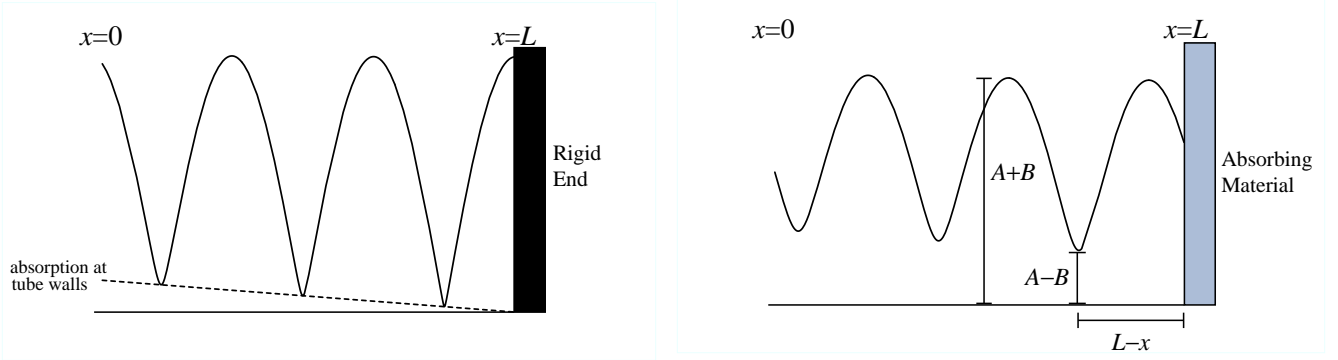$$\mathbf{Z}_{mL} = \rho c S\left[\frac{1+\frac{B}{A}e^{i\theta}}{1-\frac{B}{A}e^{i\theta}}\right] \ . \qquad (6)$$

Thus, given the ratio of incident to reflected amplitudes, and the phase shift θ, the mechanical impedance of the sample may be determined.

After a little algebraic manipulation, the pressure amplitude $P=|\mathbf{p}|$ of the wave from Eq. (2) may be written as

$$P = |\mathbf{p}| = \left\{(A+B)^2\cos^2\left[k\,(L-x) - \theta/2\right] + (A-B)^2\sin^2\left[k\,(L-x) - \theta/2\right]\right\}^{1/2} \qquad (7)$$

This pressure amplitude is shown in the figures at the top of the next page. The figure on the left shows the pressure amplitude in the pipe with a rigid termination at $x=L$. All of the sound energy incident upon the termination is reflected with the same amplitude. However, there may be some absorption along the walls as the waves travel back and forth along the pipe. The figure on the right represents the case when the pipe is terminated at $x=L$ with some acoustic absorbing material. Now some of the incident sound energy is absorbed by the material so that the reflected waves do

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[†] For a circular waveguide (pipe) filled with air, the highest frequency at which only plane waves will propagate is given by $f_{\max} \approx 101/a$ where a is the radius of the waveguide. For the 10 cm diameter pipe used in this experiment $f_{\max} \approx 2000\,\text{Hz}$.

not have the same amplitude as incident waves. In addition the absorbing material introduces a phase shift upon reflection. The amplitude at a pressure antinode (maximum pressure) is $(A+B)$, and the amplitude at a pressure node (minimum pressure) is $(A-B)$. It is not possible to measure A or B directly. However, we can measure $(A+B)$ and $(A-B)$ using the standing wave tube.

We define the ratio of pressure maximum to pressure minimum as the *standing wave ratio*

$$SWR = \frac{A+B}{A-B} \tag{8}$$

which may be arranged to provide the reflection coefficient B/A and the sound power reflection coefficient R_Π

$$\frac{B}{A} = \frac{SWR-1}{SWR+1}, \qquad R_\Pi = \left|\frac{B}{A}\right|^2 = \left(\frac{SWR-1}{SWR+1}\right)^2. \tag{9}$$

A pressure minimum occurs when

$$\cos\left[k\left(L-x\right)-\theta/2\right]=0 \qquad \text{and} \qquad \sin\left[k\left(L-x\right)-\theta/2\right]=1 \tag{10}$$

which requires that

$$k\left(L-x\right)-\theta/2 = \left(n-\frac{1}{2}\right)\pi \tag{11}$$

or

$$\theta = 2\frac{\omega}{c}\left(L-x\right)-\left(2n-1\right)\pi \tag{12}$$

where the quantity $(L-x)$ equals the distance from the test sample to the first pressure minimum $(n=1)$ as shown in the figure. The B&K Standing Wave Tube Apparatus allows for measurement of the maximum pressure, $(A+B)$, the minimum pressure, $(A-B)$, and the distance from the sample of the first minimum, $(L-x)$.

The complex mechanical impedance of the test sample may then obtained by substituting (12) and (9) into (6). The mechanical impedance of the test sample may be a complicated function of frequency, and it may be necessary to repeat the above measurements over the range of frequencies of interest. When a large number of frequencies are measured, it is inconvenient and unnecessary to carry out all the calculations as outlined above. Instead, use of a Smith calculator, (see reference 5) a nomographic chart, enables rapid determination of the real and imaginary parts of the impedance directly from measurements of the standing wave ratio and the position of the node nearest the sample.

The sound power absorption coefficient α for the test sample at a given frequency is given by

$$\alpha = 1 - R_\Pi^2 = 1 - \frac{(SWR-1)^2}{(SWR+1)^2} = \frac{4}{SWR+\frac{1}{SWR}+2}. \tag{13}$$

As was the case for the impedance, the absorption coefficient may be a function of frequency, and measurements over the frequency range of interest may be required.

Example Calculation of Impedance and Absorption Coefficient

When the standing wave tube is driven at a frequency of 1000 Hz with some test sample at the end of the impedance tube, the standing wave ratio is $SWR = 2$ and the first node is 0.129m from the sample end. Then $(L - x) = 0.129\,\mathrm{m}$ and $\theta = 2(2\pi 1000/343)(0.129) - \pi = 1.58 \approx \pi/2$. Furthermore, $B/A = (2 - 1)/(2 + 1) = 1/3$ so that

$$\frac{\mathbf{Z}_{mL}}{\rho c S} = \left[\frac{1 + \frac{1}{3}e^{i\,\pi/2}}{1 - \frac{1}{3}e^{i\,\pi/2}}\right] = 0.80 + i\,0.60 \qquad \text{and} \qquad \alpha = 1 - \left(\frac{B}{A}\right)^2 = 1 - \frac{1}{9} = 0.89,$$

which means that 89% of the incident sound power is absorbed by the sample.

III. Equipment

- Function/Waveform Generator (Hewlett-Packard 33120)
- Audio amplifier
- Several samples of acoustic absorbing material
- B&K Real-Time Frequency Analyzer Type 2133 (or equivalent 1/3 Octave Band Analyzer)
- B&K Frequency Analyzer Type 2107.
- Brüel & Kjær Standing Wave Apparatus Type 4002

The complete impedance tube measurement apparatus consists of two measuring tubes: a large one with an interior diameter of 10 cm, covering the frequency range from 90 to 2000 Hz, and a smaller tube with a diameter of 3 cm, covering the frequency range from 800 to 6500 Hz. A set of three sample holders is supplied for each tube. The tube is screwed to the loudspeaker cabinet. The microphone probes are led through a hold in the loudspeaker, and are connected at the back of the cabinet with the microphone carriage, which runs on a track provided with a scale. The actual microphone lies in elastic mountings within the microphone carriage, well insulated from airborne noise and impact sound or vibrations.

IV. Procedure and Analysis

A. Calculating α and Z_{mL} using an octave band frequency analyzer

- Place a sample of acoustic absorbing material in the cap of the impedance tube, and clamp the cap down tight. **<u>Be sure to record the type of absorbing material</u>**.
- Power up the frequency analyzer and set it up for octave band measurements.
- Connect the microphone jack to the Direct input on the Frequency Analyzer. You will have to use alligator clips and banana plugs to make this connection. Connect the HP function generator through the amplifier to the loudspeaker. Turn on the function generator and set the amplifier volume to about 3.
- ☐ Set the function generator to produce a 125 Hz sine wave.
- Press the [Start] key on the frequency analyzer to begin taking a measurement.
- Set the cursor on the Frequency Analyzer to the 125 Hz band.
- Starting with the microphone car close to the loudspeaker (this positions the microphone probe close to the sample — *be careful not to push the microphone probe into the sample*), move the car to the right until the cursor on the frequency analyzer indicates a minimum level for the 125 Hz band. Record the minimum level (in dB) for the 125 Hz band.
- Locate and record (using the ruled scale under the microphone car) the position of the first minimum from the sample end of the tube.
- Move the microphone car until the cursor reading indicates a maximum level for the 125 Hz band. Record the maximum level (in dB) for the 125 Hz band.
- Return to step ☐ and repeat for the 250 Hz, 500 Hz, 1000 Hz and 2000 Hz octave bands.

B. Analysis of α and Z_{mL} calculations

For each octave band (making a table might be a good idea):

○ Calculate the maximum and minimum rms pressures from the recorded decibel levels using

$$Level_{\text{dB}} = 20 \log p_{\text{rms}} \qquad \Longrightarrow \qquad p_{\text{rms}} = 10^{(Level_{\text{dB}}/20)}$$

○ Calculate the standing wave ratio SWR using

$$SWR = \frac{p_{\text{rms,max}}}{p_{\text{rms,min}}}$$

○ Calculate the sound power reflection (B/A) coefficient using Eq. (9).
○ Calculate the sound power absorption coefficient α using Eq. (13).
○ Calculate the phase θ from Eq. (12) with $n = 1$ and with the quantity $(L - x)$ equal to the position of the first minimum.
○ Calculate the normalized mechanical impedance $Z_{mL}/\rho c S$ of the test sample using Eq. (6)
○ Calculate the mechanical impedance using the Smith Chart.

C. Direct Measurement of α using the B&K Frequency Analyzer Type 2107

The B&K Frequency Analyzer Type 2107 is designed especially for use with the Standing Wave Apparatus Type 4002, so that the sound power absorption coefficients may be measured directly without calculation.

• Keep the same test sample in the impedance tube.
• Disconnect the microphone from the B&K Real-Time Frequency Analyzer Type 2133 and turn the analyzer off.
• Connect the microphone to the "Amplifier Input" jack on the Frequency Analyzer Type 2107.
□ Set the function generator to produce a 125 Hz sine wave.
• Using the "Frequency Tuning" dial and the "Frequency Range" knob, set the frequency of the analyzer to 125 Hz.
• Set the "Input Potentiometer" knob to about 5, and adjust the "Meter Range" and "Range Multiplier" knobs until the needle on the "Indicating Meter" shows a response somewhere in the middle of the scale.
• While watching the needle, move the microphone car until you find a pressure maximum (don't worry about the exact value indicated by the needle on the scale).
• Adjust the "Input Potentiometer" knob so that the needle reads 100% on the top line of the bottom scale.
• Move the microphone car until you find the pressure minimum closest to the sample.
○ Read off the percent from the top line of the bottom scale as indicated by the needle. This percent (converted to a decimal: 86%→0.86) is the sound power absorption coefficient.
○ For this sample, compare these measured values of the sound power absorption coefficient to the calculations previously obtained. Except for the lowest frequency band (125 Hz), the measured and calculated values should agree closely.
△ Return to step □ and repeat for the 250 Hz, 500 Hz, 1000 Hz and 2000 Hz octave bands..
• Repeat steps □ through △ for TWO additional samples. Be sure to record a description of the samples you are measuring.

D. Analysis of α measurements

⇒ Compare calculated and measured values for the absorption coefficient of your first sample.
⇒ Plot the absorption coefficient as a function of frequency for each of the three different samples. You can plot results of all three measurements on the same graph for comparison.
⇒ Some suggestions for analysis of your results: How does the absorption vary with frequency? Which samples provide the greatest amount of absorption? Over what frequency ranges are the samples most effective?

V. References

1. "Instructions and Applications" for Standing Wave Apparatus Type 4002 and Frequency Analyzer Type 2107, (Brüel & Kjær, 1967).

3. Kinsler, Frey, Coppens, and Sanders, *Fundamentals of Acoustics*, Third Edition, (John Wiley & Sons, 1982), Chapter 9.

2. Reynolds, *Engineering Principles of Acoustics: Noise and Vibration Control*, (Allyn & Bacon, 1981).

4. Bies and Hansen, *Engineering Noise Control: Theory and Practice*, Second Edition, (Chapman & Hall, 1996), Appendix D.

5. Elmore and Heald, *Physics of Waves*, (Dover reprint, 1985), Appendix B.

6. A very nice PostScript copy of a Smith Chart may be downloaded from the WWW URL: `http://marconi.uakron.edu/ wtm/smith.html`