Testing the Nonlinearity of Piano Hammers Using Residual Shock Spectra

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Summary
Force pulses and residual shock spectra of voiced, unvoiced (soft), and used (hard) piano hammers are compared. The peak frequency $f_{\max}$ of the residual shock spectrum is related to the frequency range over which the hammer will be most effective in exciting string modes. Hammer speeds of 1 to 6 m/s, used in these experiments, span the normal dynamic range of the piano. Peak force is related to pulse duration and also to a nonlinearity exponent in the equation relating force to compression of the felt. For lower notes on the piano, $f_{\max}$ is well above the fundamental frequency which helps to explain the dominance of higher partials in the bass notes. At the treble end, however, $f_{\max}$ is comparable to the fundamental frequency, resulting in a strong fundamental and few partials in these notes on a piano. In addition to its usefulness in piano research, the residual shock spectrum could serve as a useful guide in the production and voicing of pianos.

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1. Introduction

Although the hammers in the very first pianos consisted of wooden heads covered with leather, felt hammers have been standard since about 1830. Modern hammers consist of a wooden molding covered with several layers of compressed wool felt, whose hardness is carefully controlled. In order to produce a good tone, the hardness has a gradient which may be adjusted using various techniques in a process called voicing.

The static hardness of a piano hammer has a great deal of influence on the resulting piano sound. Hard hammers are better at exciting high frequency modes of a piano string's vibration so that the resulting tone quality may be characterized as being bright, tinny, or harsh. Soft hammers, on the other hand, do not excite high frequencies very well, and the resulting tone is somewhat dull or dark. The static hardness of a piano hammer may be tested by a durometer or hardness tester [1]. In a typical piano, treble hammers are much harder than bass hammers.

The dynamic hardness of a piano hammer also plays an important role in the final piano sound. A piano hammer behaves somewhat as a hardening spring; for large impact forces the hammer felt appears harder than it does for low impact forces. In the piano this means that a loud note sounds much brighter (i.e. contains more high frequencies) than a quiet note. It is difficult to test the dynamic hardness of hammers except by listening to them in a finished piano. If the piano tone is not as desired then the hammer hardness must be adjusted by voicing. This paper presents a method of testing the dynamic hardness of a piano hammer using the residual shock spectrum.

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2. Static and dynamic measurements of hammer felt hardness

Static measurements of applied force versus compression of wool felt pads [2, 3] do not obey Hooke's Law, but display a nonlinear relationship as for a hardening spring. Also evident in force-compression measurements of felt pads is a hysteresis effect, due largely to slippage of the wool fibers which occurs during slow and large amplitude compression [4]. Since piano hammers are covered with layer(s) of wool felt, it is not surprising that nonlinear force-compression curves and hysteresis loops are also characteristic of piano hammers.

Static measurements by Hall and Askenfelt [5] of force-compression curves for several piano hammers can be fit to a power law:

$$F = K\xi^p, \tag{1}$$

where K has units of $\mathrm{N/m}^p$ and is a generalized stiffness of the hammer, and the exponent p describes how much the stiffness changes with force. Hall and Askenfelt measured values of p ranging from 2.2 to 3.5 for hammers taken from pianos, and 1.5 to 2.8 for unused hammers. A greater value of p means a greater range of hammer stiffness for a given range of force.

Dynamic measurements of the force and felt compression, observed during the impact between a piano hammer and a rigidly fixed string, have been obtained by Suzuki [6, 7], Boutillon [8], and Yanagisawa and Nakamura [9]. Data may be fit rather well to Eq.(1), with typical values of p ranging from 2.3 to 3.6 for voiced piano hammers. The measurements of Boutillon and Yanagisawa and Nakamura clearly indicate hysteresis loops, with the coefficient K and exponent p having different values for compression and relaxation.

In addition to their static measurements, Hall and Askenfelt [5, 10] also devised a dynamic method of measuring the nonlinearity in the hammer compliance. Instead of measuring the hammer felt compression, they used an oscilloscope to display the force-time pulse shape of the hammer as measured by the force transducer. The contact duration τ for a blow on a rigid surface is related to the maximum force by

$$\tau \propto (F_{\max})^{(1-p)/2p}, \tag{2}$$

where the exponent p is the same as in Eq.(1). Measurements made for voiced hammers from several pianos show a smooth increase from $p \simeq 2$ in the bass, to $p \simeq 4$ in the treble. A set of four matched hammers whose hardness was adjusted in various degrees from very hard to very soft, yielded $p = 2.3$ for the hardest hammer, $p = 2.8$ for the softest hammer, and $p = 3.3$ for a hammer so soft it was effectively ruined.

While the measurements mentioned above provide information about the dynamic hardness of hammer felt, they do not directly provide information regarding how the dynamic hardness of a hammer affects the string vibrations and ultimately the piano sound. A dynamic method providing this information is needed.

Bork [11] had measured the acoustical properties of percussion mallets and applied a dynamic method, using the residual shock spectrum, [12] to determine the range of frequencies that a given mallet moat effectively excites for a given range of impact velocities. He found that the peak value of the residual shock spectrum indicates the frequency range over which a mallet is most effective for a given blow, and that this peak value varies with blow strength. The present authors have applied this dynamic method to study the nonlinear behavior of piano hammers.

3. The residual shock spectrum

3.1. Shocks and shock spectra

A shock may be defined as the "transmission of kinetic energy to a system, the duration of the energy transfer being short compared to the natural period of oscillation of the system" [13]. A shock pulse is a time history described in terms of force, acceleration, velocity, or displacement. The usual goal of shock analysis is to estimate the effect of a shock on a certain mechanical system. The shock spectrum, or response spectrum, describes the response of the system to the applied shock; the response may be given in terms of acceleration, velocity, or displacement. The shock spectrum may be measured by applying a shock to a series of linear, undamped, single degree-of-freedom systems and plotting the maximum response of each system versus its resonance frequency. There are three basic forms of the shock spectrum: the *initial shock spectrum* is the maximum response of the system while the shock is still acting; the *residual shock spectrum* is the maximum response after the shock has stopped; the *maximax spectrum* is the maximum response over all time [13, 14].

If the shock pulse is of a type that can be expressed in simple mathematical terms, then the shock spectrum may be calculated [15]. The shape and duration of the shock pulse determine the shape and frequency range of the shock spectrum.

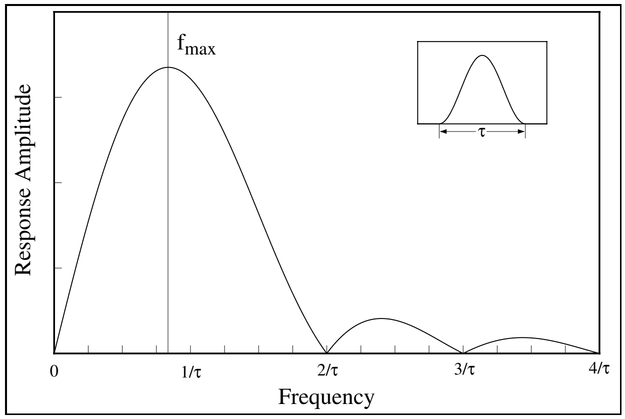

Figure 1. Residual shock spectrum calculated for a sine-squared pulse of duration τ.

Figure 1 shows the residual shock spectrum for a sine-squared pulse of duration τ. The frequency $f_{\max}$ corresponding to the center of the main peak is the frequency at which a single degree-of-freedom system would be most effectively excited by this specific shock pulse. If the system to which the shock is to be applied is a complex system, with many resonance frequencies, a resonant mode with a frequency near $f_{\max}$ will be maximally excited, whereas a mode with a frequency much lower or higher than $f_{\max}$ will be only poorly excited. With a prior knowledge of the resonance frequencies of a system, the residual shock spectrum may be used to predict how effectively each of those resonances will be excited by an applied shock pulse.

The residual shock spectrum has a simple relationship to the Fourier transform [13, 16]. The maximum acceleration response to an acceleration shock is related to the Fourier spectrum of the shock pulse by

$$R_a(\omega) = \omega |F_a(j\omega)| \ , \tag{3}$$

where $R_a(\omega)$ is the acceleration residual shock spectrum and $F_a(\omega)$ is the Fourier spectrum of the acceleration shock pulse. If the shock is a force pulse then the acceleration response depends on the mass M of the system to which it is applied [11,12]

$$R_a(\omega) = \frac{\omega}{M} |F(j\omega)| \ , \tag{4}$$

where in this case $F(\omega)$ is the Fourier spectrum of the force pulse. If the mass of the system is constant then a measurement of $\omega|F(\omega)|$ may still be considered to be a measure of the acceleration amplitude. Equation (3) allows the residual shock spectrum to be obtained for a shock pulse that may be measured, but not expressed mathematically. If a piano hammer strikes a stationary force transducer, the Fourier transform of the force pulse and the resulting acceleration residual shock spectrum may both be obtained by means of a fast-Fourier transform (FFT) analyzer.

3.2. Application to piano hammers

In our application of shock theory the residual shock spectrum of a piano hammer as it strikes a rigidly mounted force transducer provides information about how the dynamic hardness of a piano hammer affects the sound produced by a piano. The hammer is regarded as providing a force shock to the string, which is treated as a complex system with many resonance frequencies. However, in such an application several things must be considered.

First of all, the interaction between the hammer and string in a real piano is a very complicated one. This interaction affects the shape of the shock pulse applied by the hammer; reflections from the near end of the string can cause valleys in the pulse shape [7, 8] so that it differs considerably from the smooth sine-squared-like pulse shape obtained when the hammer hits a rigid object. Also, there are often multiple contacts between hammer and string before the hammer is thrown sufficiently away from the string [17, 18, 19]. Secondly, the definition of a shock requires a duration shorter than the natural period of the receiving system. For bass hammers, the hammer-string contact time is only a fraction of the period of the string fundamental, and in the middle register it is about half the period [20, 21]. However, in the treble register the hammer-string contact time is several periods. Thirdly, the vibration of the hammer shank may also affect the interaction between hammer and string [22, 23], since the force of the hammer on the string is not constant when the shank is vibrating.

These problems must be considered when interpreting the residual shock spectrum of a piano hammer obtained as the hammer strikes a rigid object and applying the results to the hammer-string problem. If, however, it is assumed that the force of a hammer hitting a string may be roughly approximated by the force of the hammer hitting a rigid object, and attention is focused on the hammer itself, then our measured residual shock spectrum provides a rough prediction of the hammer's ability to transfer its kinetic energy to the string. The residual shock spectrum serves as a "fingerprint" for the hammer giving an approximate frequency range over which the hammer is most effective in exciting string modes. The shock spectrum also shows how this frequency range changes with hammer size, velocity, and felt stiffness.

Not only might such information increase our understanding of the nonlinear behavior of the hammer, but the residual shock spectrum of a piano hammer might also prove to be a useful diagnostic tool in the piano industry. Preliminary voicing of hammers could be done even before they are placed in a piano. Measuring the residual shock spectrum is a quick process; it could be implemented to test the quality of hammers in cheaper pianos which are usually not voiced at all.

4. Experimental method

4.1. Force impulse and shock spectrum

A falling pendulum was used to apply a specified impulse to the piano key, the force being transferred through the action to throw the hammer upwards, as shown in Figure 2. The upward-swinging piano hammer struck the smooth, flat head of a screw (9 mm in diameter) attached to a Brüel & Kjær 8001 impedance head rigidly mounted in a lead brick support (with approximate mass of 15 kg) , as shown in Figure 2. The force transducer was positioned so that the "let-off" distance between the hammer and the screw head was approximately 3 mm, which is about the same value as the let-off distance between the hammer and a bass string in a grand piano. The screw head was flat to provide uniformity for all hammers measured, as well as to prevent the hammer felt developing grooves rubs against the strings, as in a piano. In preliminary measurements, a 1 cm length of thick, double-wound piano wire was used, but there was no noticeable difference in the results when the flat screw was used instead.

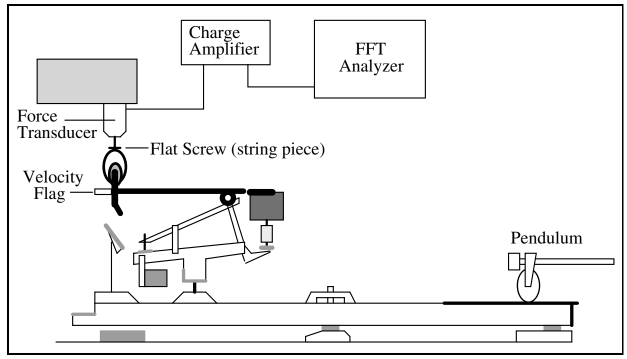

Figure 2. Experimental setup. The falling pendulum applies an impulse to the piano key, which throws the hammer upward to strike the force transducer. The velocity flag passes through a photogate in order to measure the hammer velocity.

The force signal from the impedance head was amplified with a Brüel & Kjær 2651 charge amplifier, and fed into an Ono Sokki CF-350 FFT analyzer. The residual shock spectrum was obtained by multiplying the power spectrum (Fourier transform of the force pulse) by ω, as per Eq.(3). The shape of the shock pulse was simultaneously recorded on a second FFT analyzer, a Nicolet Scientific Corporation 660A, in order to measure pulse width and peak force. Experimental values of peak force covered the range of 5–310 N, which compares well with values of 2–300 N reported by Hall and Askenfelt [5].

4.2. Hammer velocity

The hammer velocity was measured with a *Precision Timer* system from Vernier Software, consisting of four photogates and software running on an 8086 computer. Two gates were used: gate 1 acted as a trigger and gate 2 was used to determine the average velocity of the hammer during the last 7.5 mm of hammer travel prior to impact. Experimental hammer velocities ranged from 0.6 m/s to 6.2 m/s. The program measured time to the nearest 0.1 ms, giving an uncertainty of ± 0.3 m/s ($\pm 6\%$) at 5 m/s, and ± 0.005 m/s ($\pm 0.8\%$) at 0.6 m/s, respectively. The measured range of velocities agrees with Boutillon [8] and Askenfelt and Jansson [23].

At velocities higher than 6.2 m/s the maximum force was greater than 300 N which was the maximum dynamic load specification of the force transducer.

4.3. The piano hammers

The main set of piano hammers for this experiment belonged to a complete grand piano action provided by Steinway & Sons Inc. (model D 274 cm). The hammers were voiced by a technician at the factory, and it appeared that they had been in the piano for awhile since many of the hammers showed slight grooves from rubbing against the piano strings.

A second set of hammers were removed from a 1972 model B Steinway (211 cm) after having been voiced and adjusted several times.[1] They were removed from the piano because they had become too hard, causing the piano to sound harsh, and many hammers in the treble region did not have enough felt left to be voiced any softer. The hammers were numbers 1, 13, 25, 37, 51, 63, 75, 86 from the keyboard, (corresponding to notes A_0, A_1, A_2, A_3, B_4, B_5, B_6, $A_7^{\#}$, respectively) and were still attached to their original shanks (complete with flange, drop screw, bushing and center pin, and hammer knuckle). This set will be referred to as "hard hammers."

A third set of hammers, in the following referred to as "soft hammers," were unfinished hammers from an unspecified Steinway grand piano. These hammers were softer than the voiced set, because the softer, looser outer layers of felt had not yet been filed off in the voicing process. The unfinished wooden tails of the hammers were carved to approximately correct shapes and glued to shanks taken from the 1972 model B Steinway mentioned above. The unvoiced set of hammers corresponded approximately to the positions 13, 37, 51, 64, 68, 73, 78, 85, 88 (A_1, A_3, B_4, C_6, E_6, A_6, D_7, A_7, C_8, respectively). Both the hard and soft hammer shanks were mounted on a Steinway single-key action model.

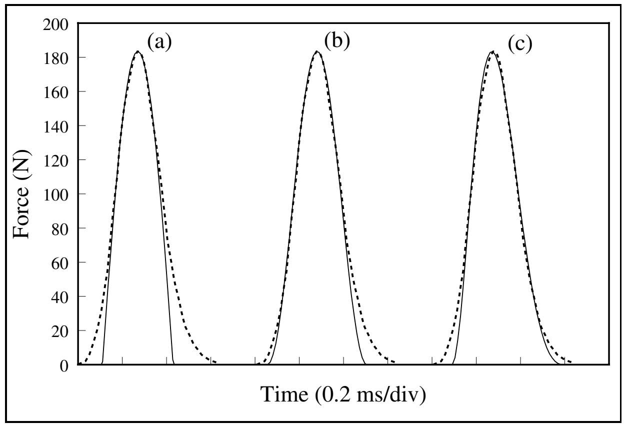

Figure 3. Shock pulse from hard A_3 hammer at a speed of 4 m/s (dashed curve) is compared with: (a) a half-sine pulse; (b) a sine-squared pulse; (c) a skewed versed-sine pulse.

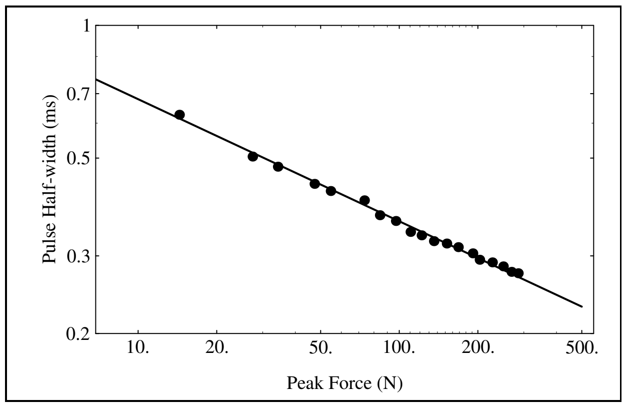

Figure 4. Pulse half-widths vs peak force for a voiced D_2 hammer from a Steinway model D piano.

5. Results

5.1. Pulse shape, duration, and peak amplitude

The force pulse recorded for hammer A_3 from the hard hammer set at a speed of 4 m/s is shown as the dashed curves in Figure 3. The maximum force is 183 N and the half-width is 0.24 ms. The asymmetry in the measured pulse is due to the hysteresis of the hammer felt. The measured pulse is compared with a half-sine pulse, a sine-squared pulse, and a skewed versed-sine pulse (the product of a decaying exponential and a sine-squared pulse) in Figure 3 (a), (b), and (c), respectively. These three shapes were chosen not only because they fit the data rather well, but also because they are shock shapes whose shock spectra are well documented in the literature.

For comparison, the impulse obtained from an experimental hammer with a polyurethane elastomer head could be very closely approximated by a half-sine pulse, indicating a linear behavior.[2] As one would expect for a linear hammer, the pulse duration remained essentially constant over the velocity range of 1-5 m/s, whereas for all real piano hammers measured in this study the pulse duration decreased with increasing velocity. For hard hammer A_3 the pulse half-width decreased from 0.42 ms at 1 m/s to 0.23 at 5 m/s.

Figure 4 is a logarithmic plot of the pulse half-width versus the peak force for the D_2 hammer. From such a plot the stiffness nonlinearity exponent p may be calculated using Eq.(2). If the slope in Figure 4 is denoted by s then $p = (2s+1)^{-1}$ [10]. For the D_2 hammer $s{=}-0.27$ corresponding to $p{=}2.2$. Nonlinearity exponents for 13 voiced hammers and 6 hard hammers are compared in Figure 5. Exponent values agree well with results of Hall and Askenfelt [5, 10]. The exponent p is not a measure of the hammer felt stiffness,

[1] The hard hammers were provided by David Graham, piano technician for the Northern Illinois University School of Music.

[2] The experimental hammer of polyurethane elastomer was supplied by Anders Askenfelt, Dept. of Speech, Music and Hearing, Royal Institute of Technology, Stockholm. This hammer had been developed in cooperation with the Dept. of Polymer Technology at the same institute.

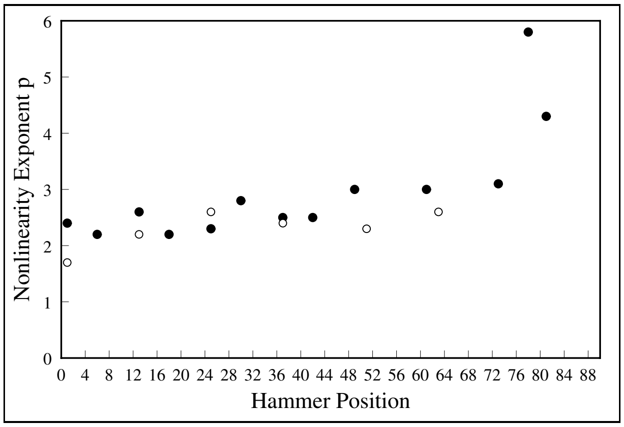

Figure 5. Effective stiffness nonlinearity exponents for 13 voiced hammers (solid circles) and 6 hard hammers (open circles).

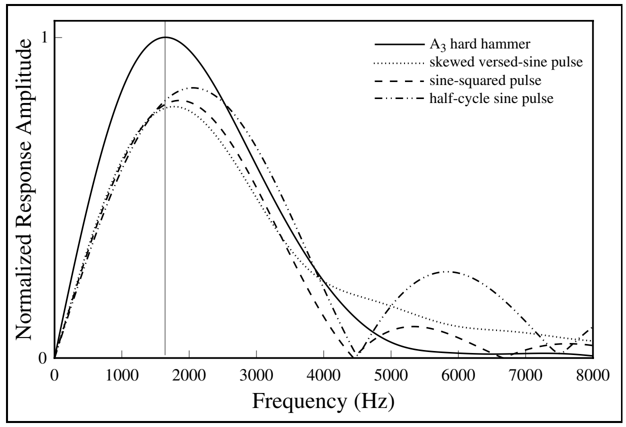

Figure 6. Residual shock spectrum of hard A_3 hammer at a speed of 4 m/s (solid curve) compared with residual shock spectra calculated for the half-sine pulse, sine-squared pulse, and skewed versed-sine pulse from Figure 3.

Table I. Values of a and b from $f_{\max} = av^b$ for voiced hammers

Hammer	a	b
1 (A_0)	763.2	0.41
6 (D_1)	732.3	0.39
13 (A_1)	639.7	0.41
18 (D_2)	746.6	0.36
25 (A_2)	723.2	0.40
30 (D_3)	749.6	0.44
37 (A_3)	755.6	0.46
42 (D_4)	849.1	0.40
49 (A_4)	827.1	0.46
61 (A_5)	992.6	0.48
73 (A_6)	1147.0	0.50
78 (D_7)	1261.9	0.64
81 (F_7)	1306.9	0.62

but rather a measure of how much the stiffness changes with force. The greater the value of p the greater the range of hammer stiffness. From Figure 5 we might predict that treble hammers will show greater changes in stiffness than bass hammers if both are tested over the same range of velocities. Figure 5 also shows that most of the hard hammers have lower exponent values than the voiced hammers, a result that agrees with Hall and Askenfelt [5, 10]. This suggests that the hard hammers should show smaller changes in stiffness than the voiced hammers.

5.2. Residual shock spectra and the peak frequency

Figure 6 shows the residual shock spectrum measured for hard hammer A_3 at 4 m/s from Figure 3 compared to the residual shock spectra calculated for the half-sine pulse, sine-squared pulse, and skewed versed-sine pulse from Figure 3 (a), (b), and (c), respectively. The value of $f_{\max}$ predicted by the skewed versed-sine pulse is about 140 Hz higher than the measured value, and the sine-squared pulse prediction is about 230 Hz to high. In both cases the amplitude is about 20% too low, but if we are only interested in the location of $f_{\max}$ for a given velocity and only able to measure the pulse shape, say with an oscilloscope, then either could be used as a rough estimate to calculate the residual shock spectrum.

Figure 7 shows the residual shock spectra for the voiced Steinway model D hammers A_0 and F_7 measured at velocities of 1 m/s and 4 m/s. The vertical line through the peak of each spectrum locates $f_{\max}$. For hammer A_0, $f_{\max}$ is 775 Hz at 1 m/s and 1370 Hz at 4 m/s. For hammer F_7, $f_{\max}$ is 1300 Hz at 1 m/s and 3038 Hz at 4 m/s. For both hammers, as the velocity increases the shock spectrum broadens and $f_{\max}$ shifts upwards in frequency. This is in direct contrast to the linear polyurethane hammer which showed no change in $f_{\max}$ as the velocity increased. The increase in frequency observed in Figure 7 is not proportional to the increase in velocity, nor is the rate of increase the same for the two hammers. For hammer A_0, $f_{\max}$ increases by a factor of 1.8 while the velocity increases by a factor of four. For hammer F_7, $f_{\max}$ increases by a factor of 2.3 while the velocity increases by a factor of four.

The relationship between $f_{\max}$ and hammer velocity v is more clearly shown in Figure 8 for hammers D_1 and A_4 from the properly voiced Steinway model D set. This plot shows very clearly that the increase in $f_{\max}$ is not proportional to velocity, and that the range of $f_{\max}$ values is not the same for both hammers. The data can be fit fairly well to an equation of the form

$$f_{\max} = av^b ,$$

where the exponent b ranges between 0.4 and 0.5 for most of the voiced Steinway hammers as shown in Table I. The value of the exponent b determines the range of $f_{\max}$ values for a given range of velocities; a larger value of b means a hammer is efficient at exciting a wider range of frequencies. There is a slight increase in values of b from bass to treble.

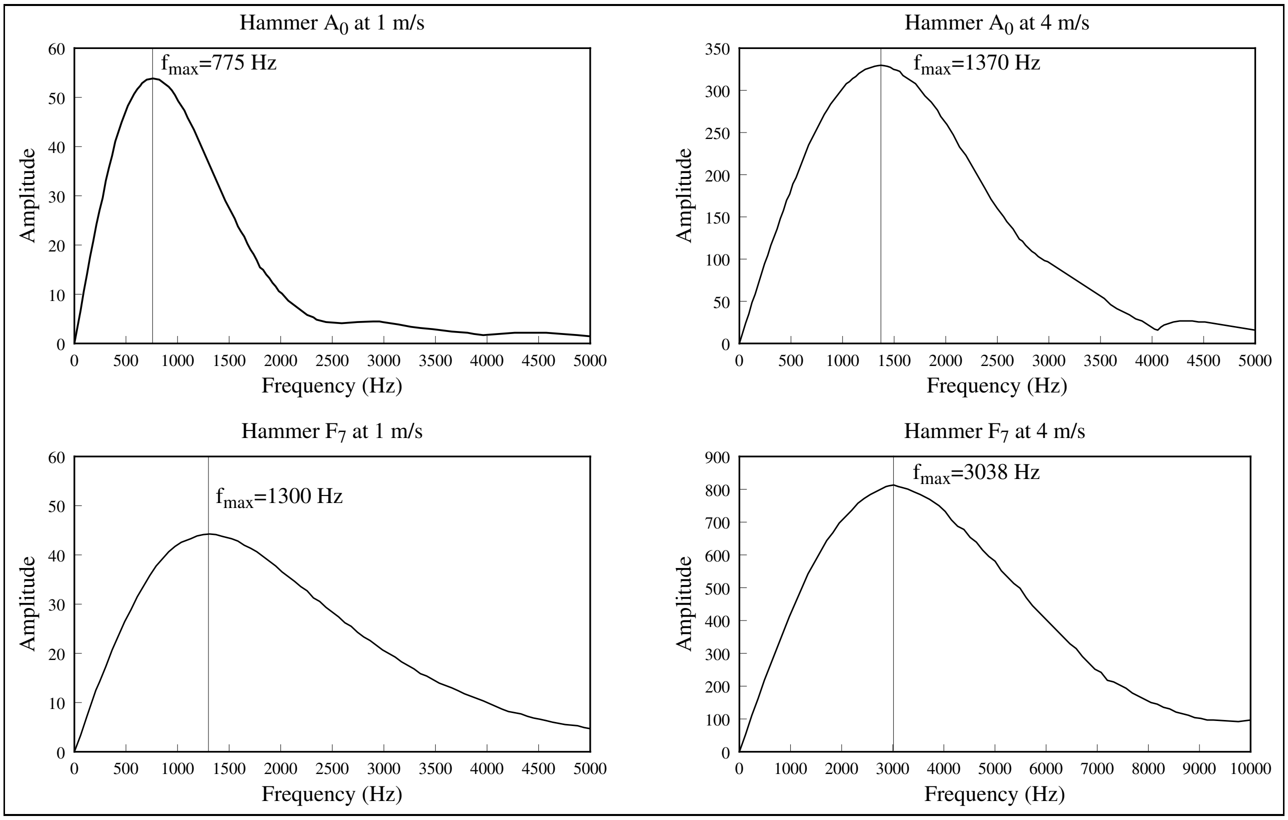

Figure 7. Residual shock spectra for voiced Steinway model D hammers A_0 and F_7 at 1 and 4 m/s.

6. Discussion of results

6.1. Residual shock spectra and the peak frequency

The broadening of the residual shock spectra and the upward shift of $f_{\max}$, apparent in Figure 7, suggests that as hammer velocity increases the hammer should become more effective at exciting the high frequency modes of the string. The fundamental frequency of the string corresponding to hammer A_0 is 27.5 Hz. However, since $f_{\max}$ ranges from 775 Hz to 1406 Hz, it appears that hammer A_0 will not be very effective at exciting the fundamental mode of its string. Rather, it would be more effective at exciting modes of vibration with frequencies between 500 Hz and 2000 Hz. The fundamental frequency of the F_7 string is 2794 Hz, which lies between the values of $f_{\max}$ shown in Figure 7. Thus, hammer F_7 should be effective in exciting the string fundamental, but less effective in exciting the higher modes of the string.

Figure 9 shows the variation of $f_{\max}$ for velocities of 1, 2, 3, 4 and 5 m/s plotted as a function of position on the keyboard of the voiced Steinway action. The curve represents the frequencies of the notes of the musical scale, starting with 27.5 Hz for A_0 and ending with 4186 Hz for C_8, the frequency doubling every octave.

Figure 9 shows how $f_{\max}$ for each hammer shifts upward in frequency as the velocity increases. It is interesting to note that the smaller treble hammers have a much greater range of $f_{\max}$ values than do the larger bass hammers; hammer #73

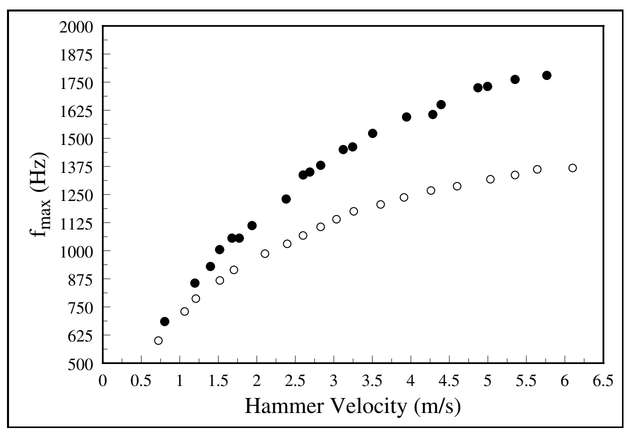

Figure 8. Peak frequency $f_{\max}$ of the residual shock spectrum vs hammer velocity for voiced hammers D_1 (white) and A_4 (black) from a Steinway model D piano.

(C_8) has a range of 1800 Hz while hammer #13 (A_0) has a range of only 650 Hz. We should recall that Figure 5 shows a general increase in the value of the stiffness nonlinearity exponent p from bass to treble. A high value of p means a wider range of hammer stiffness, resulting in a range of pulse durations and a corresponding wider range of peak frequencies in the residual shock spectra.

It is also interesting to note, from Figure 9 that for any given velocity $f_{\max}$ does not double with each octave along

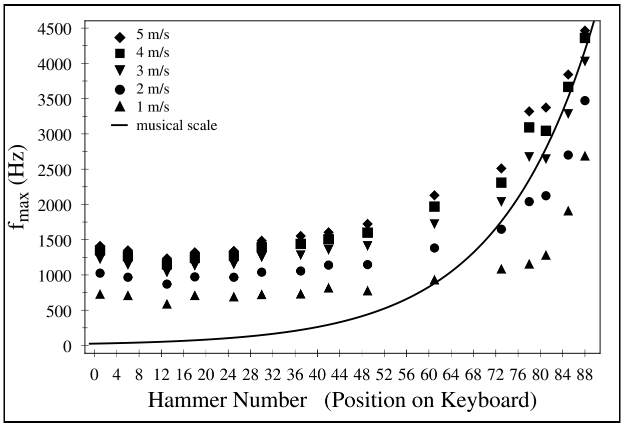

Figure 9. Peak frequencies of residual shock spectra for velocities of 1, 2, 3, 4, and 5 m/s for 15 voiced hammers from a Steinway model D piano. Solid curve gives the fundamental frequencies of the 88 notes on the piano keyboard.

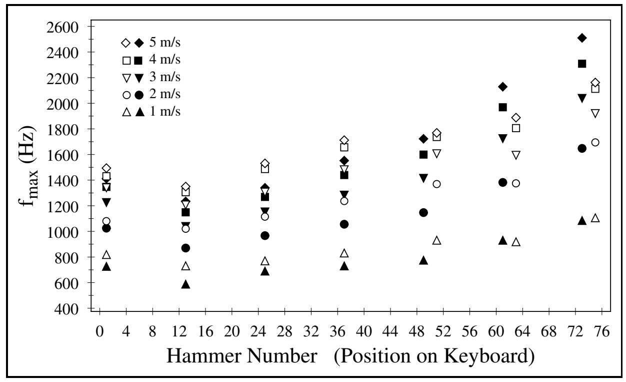

Figure 10. Comparison of peak frequencies of residual shock spectrum for hard hammers (open symbols) from a Steinway model B (211 cm) with a set of properly voiced hammers (filled symbols) from a Steinway model D (274 cm).

the keyboard. In the bass region, the values of $f_{\max}$ are much higher than the fundamental frequency, while in the treble region the values of $f_{\max}$ surround the fundamental.

Do these trends help us understand piano tone production? In the lower two octaves of the piano keyboard, the sound spectra show many high partials, the fundamental being noticeably weaker than the strongest partials (?). Sound spectra for the A_0 string extend up to about 3000 Hz and contain up to 50 partials; the fundamental is as much as 25 dB lower than the strongest partial. Toward the middle of the piano, the fundamental gains prominence, and though the upper frequency limit of the spectrum increases, the number of partials decreases. In the upper two octaves, the fundamentals completely dominate the sound spectra, and while the spectra extend up to about 10 kHz, only one or two partials are present. The role played by the piano soundboard in these measurements of radiated sound spectra must be considered, but Askenfelt and Jansson have shown similar results for spectral measurements of the strings alone [21].

The data in Fig. 9 can explain this behavior of the piano sound spectra. Let the range of values for $f_{\max}$ between velocities of 1 and 5 m/s define an "effective hammer frequency range," meaning that approximate frequency range over which the hammer is capable of exciting string modes most effectively. In the lower two octaves there is a large gap between the string fundamental frequency and the effective frequency range of the hammer, so that the hammer would be most effective at exciting the high frequency modes of the string, and less effective at exciting the fundamental. This quality of the hammer, in addition to the fact that sound radiation from the piano soundboard increases with frequency, helps to explain the dominance of high-frequency partials in the piano sound spectrum of the lower octaves.

In the middle of the keyboard, with fundamental string frequency, the gap between the effective hammer frequency range and the string fundamental narrows. In addition, the frequencies of the higher modes of string vibration are increasing, though the hammer's effective frequency range stays fairly constant. For these middle octaves, the hammer appears to be less effective at exciting the high partials and increasingly effective at exciting the fundamental frequency.

In the upper two octaves, the fundamental frequency of the string is near the center of the effective frequency range of the hammer, so that the hammer is able to strongly excite the fundamental, but only one or two partials. For the treble hammers in a piano, however, the contact time is much longer than the fundamental period and the absence of higher frequencies is also a result of their being damped out by the hammer before the string throws it away.

Having the hammer hit a fixed force transducer is a first approximation to what actually occurs in a piano when the hammer strikes the strings. However, we can use the trends in $f_{\max}$ to approximately predict the ability of the hammer to excite different modes of vibration in the string. The picture of the hammer-string excitation given in Figure 9 seems very reasonable in comparison to what is actually observed in a real piano.

6.2. Residual shock spectrum and hammer hardness

Since the residual shock spectrum seems to work very well in predicting the ability of the hammer to excite the string modes of vibration, one could also use it to try to predict the behavior of hard and soft hammers. Figure 10 shows the values of $f_{\max}$ for the set of hard hammers at velocities of 1, 2, 3, 4 and 5 m/s plotted as a function of hammer position in the keyboard, and compared to the properly voiced hammers. Over the lower two-thirds of the keyboard, the values of $f_{\max}$ at a given velocity are higher for the hard hammers than for the properly voiced hammers while the range of $f_{\max}$ values is approximately the same for both sets. The exception for hard hammers 63 and 75 for velocities of 3-5 m/s are most likely due to the fact that these hammers were very worn and the condition of the felt was rather poor.

These trends may be explained using the results shown in Figure 5 and Eq.(1). The stiffness, or hardness, of the hammer, governed by K, determines the location of $f_{\max}$; a harder hammer will have a higher $f_{\max}$. This is demonstrated in Figure 10. The variation of stiffness with applied force, or velocity, is governed by the exponent p. Larger values of p result in a larger range of $f_{\max}$ values. Figure 5 shows that over the lower two-thirds of the piano, both hard and voiced hammers have similar nonlinearity exponents, while the voiced exponents for treble hammers have a much larger value of p. Comparison of $f_{\max}$ ranges in Figure 10 shows effect of this exponent behavior.

Ignoring the damaged treble hard hammers, one can conclude that the higher values of $f_{\max}$ for hard hammers suggest that harder hammers would be better at exciting higher frequency string vibrations than would a properly voiced hammer. Indeed, the sound produced by a hard hammer is "brighter" or "harsher" than that produced by normal hammers. Measurements by Askenfelt and Jansson show that string spectra resulting from a blow by a hard hammer contain more partials than do string spectra resulting from a blow by a soft hammer [21].

Figure 11 shows the $f_{\max}$ values for the unvoiced set of soft hammers compared to the properly voiced hammers. The looser, outer layers of felt have not been removed from the unvoiced hammers, so that they are softer than the voiced hammers. The values of $f_{\max}$ at a given velocity are lower for the unvoiced hammers than for the properly voiced hammers. In addition, the ranges of $f_{\max}$ values are narrower for the unvoiced hammers. Unfortunately, values of the nonlinear exponent p were not obtained for the unvoiced hammers, so that conclusions concerning the relationship between Eq.(1) and Figure 11 may only be suggested. Soft hammers would not be expected to excite many higher frequencies in the string vibration, and the envelope of the string spectra would not change greatly for a wide range of hammer velocities. Measurements by Askenfelt and Jansson [21] show that the string spectrum produced by an overly-soft hammer is weak in high harmonics and the resulting sound is often "dull." Further experimentation is needed to completely understand the relationship between a hammer's stiffness K, nonlinear exponent p, and residual shock spectrum maximum frequency $f_{\max}$.

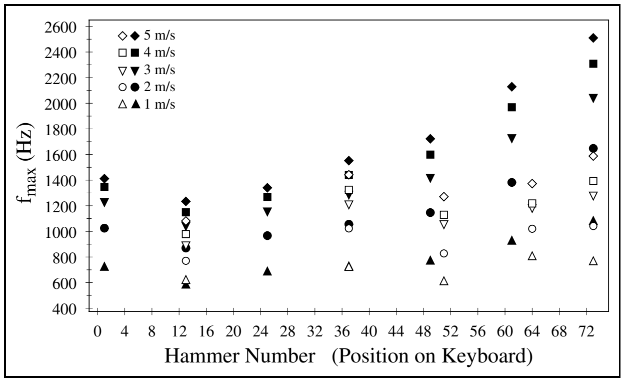

Figure 11. Comparison of peak frequencies of residual shock spectrum for unvoiced soft hammers (open symbols) from an unspecified Steinway grand piano with a set of properly voiced hammers (filled symbols) from a Steinway model D (274 cm).

7. Conclusions

The residual shock spectrum has been shown to be a useful tool for both measuring the hammer nonlinearity and for predicting the frequency range of string modes a hammer is most likely to excite. The frequency $f_{\max}$, corresponding to the peak of the residual shock spectrum, represents the frequency at which a piano hammer is most capable of exciting a string mode. For a given hammer, as the hammer velocity increases, the measured values of $f_{\max}$ also increases, though not proportionally. This helps to explain how a harder key strike (higher hammer velocity) produces a brighter tone than does a soft strike (lower hammer velocity). Measurements of $f_{\max}$ were obtained for several velocities, for 15 notes across the entire piano keyboard defining "effective hammer frequency ranges." In the bass, where string spectra show a weak fundamental and many harmonics, the effective frequency range of the hammer lay significantly higher above the string fundamental, suggesting that the hammer would weakly excite the fundamental while strongly exciting many higher harmonics. In the middle range, where string spectra show stronger fundamentals and fewer harmonics, the effective range of the hammer was still higher than the fundamental, but not nearly as much as for the bass region. In the treble, where string spectra show a dominant fundamental with very few harmonics, the effective hammer range included the fundamental, suggesting that a treble hammer would strongly excite the fundamental, but weakly excite only one or two harmonics.

Measurements for hard and soft hammers also agree well with what is observed in the piano sound. The center of the effective hammer range for the set of hard hammers was higher than for the properly voiced hammers, suggesting that hard hammers would excite more high harmonics. The unvoiced, soft hammers had $f_{\max}$ values which were lower than the properly voiced hammers, suggesting that softer hammers would excite fewer high harmonics.

In addition to its usefulness in piano research, the residual shock spectrum could serve as a useful guide in the production and voicing of piano hammers. Prior to being placed in a piano action, hammers could be tested for hardness, for their stiffness nonlinearity exponent, and for their effective frequency range.

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