The Physics of the Trampoline Effect in Baseball and Softball Bats

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ABSTRACT: In the high-speed collision between a baseball and bat, most of the initial center-of-mass kinetic energy is converted into compressional energy in the ball, and about 75% of that energy is dissipated. Some of the energy is stored in vibrational modes of the bat, particularly in the so-called "hoop modes", the most important of which is a radial deformation with a quadrupole azimuthal dependence. The lowest such mode has a frequency in the 1-3 kHz range and is strongly excited during the collision by the local compression of the shell of the bat at the point of impact. Some of the collision energy that would otherwise have been stored and mostly dissipated in the ball is stored in this mode. Interestingly and for reasons examined in this paper, much of this stored energy is returned to the ball, resulting in less overall energy dissipated and a correspondingly larger ball exit speed. This is popularly called the "trampoline effect", and the goal of this paper is to examine the physics behind the effect. A simple picture of the trampoline effect is presented and the consequences of this picture are interpreted in physical terms. Results of a more realistic model are given, along with comparisons with data. Finally a discussion of whether "corking" a wood bat produces a trampoline effect is presented.

INTRODUCTION

The collision between a baseball or softball and a bat is violent, with peak forces in the thousands of pounds required to reverse the direction of the ball in a time of order 1 ms. During the collision, the ball compresses to a fraction of its undistorted radius, comes to a momentary halt, reverses direction and then expands to its original shape. This process is inherently inefficient, with a large fraction of the original kinetic energy dissipated in the internal structure of the ball. This inefficiency is characterized by the "coefficient of restitution" (COR or *e*). For a two-body collision, the COR is defined as the ratio of the relative velocity after the collision to that before the collision. It is straightforward to show that the fraction of the original energy in the center of mass

(CM) frame that is dissipated in the collision is $1-e^2$. For a perfectly elastic collision, no energy is dissipated, the two bodies recede with the initial relative velocity, and $e=1$. For a perfectly inelastic collision, the two bodies stick together, all the initial CM energy is dissipated, and $e=0$.

The COR is actually a joint property of the two colliding bodies. Nevertheless, it is common to refer to the "COR of the ball", denoted herein by the symbol e_0, which is the COR when the ball collides with a massive rigid wall. In such a collision, all of the energy losses come from dissipation in the ball. For a baseball or softball at speeds typical of the game, e_0 is about 0.5, so that 75% of the initial energy stored in the ball is dissipated. For the collision of a baseball with a bat, e is generally different from e_0 due to the flexibility of the bat. As a result the ball and the bat mutually compress each other during the collision, so that some of the CM energy that might otherwise have gone into compressing the ball instead goes into compressing the bat. Therefore less energy gets stored and dissipated in the ball. Whether the COR increases or decreases relative to e_0 depends on how effectively the compressional energy stored in the bat is returned to the ball. For solid wood bats, the energy stored in the bat is not effectively returned to the ball but instead appears as low-frequency bending vibrations. Therefore, e never exceeds e_0, and for collisions far from the nodes of the lowest few vibrations, e is considerably less than e_0 (Nathan, 2000). For hollow bats, such as the commonly used aluminum bat, energy is also stored in the so-called hoop modes, which correspond to a compression of the thin shell. For reasons that will be examined in this paper, the energy stored in the hoop modes is efficiently returned to the ball, resulting in a COR which is larger than e_0. This phenomenon is commonly referred to as the trampoline effect. The question as to why the hoop modes are effective and the bending modes are not effective at returning stored energy to the ball is one that will be addressed in this paper.

A SIMPLE PHYSICAL PICTURE

The ball-bat collision is a complicated problem that is neither easy to solve from first principles nor particularly illuminating to do so . Therefore a toy model is proposed which, while highly simplified, captures the essential physics of the trampoline effect. The model, shown schematically in Fig. 1, is similar to that considered earlier for tennis (Cross, 2000) or baseball (Naruo and Sato, 1997) and consists of representing the ball and bat as linear springs that can mutually compress each other. The

Figure 1 *Mass-spring model for the ball-bat collision.*

ball consists of a mass m attached to a damped spring of force constant k_{ball}, with the other end of the spring free. The bat consists of a mass M attached to an undamped spring of force constant k_{bat}, with the opposite side of the spring attached to a massive wall. The collision consists of the free end of the ball spring, initially moving with speed v_o, colliding with the mass M, initially at rest. The collision is tracked numerically until the ball spring and bat mass separate, whereupon the ball has a final speed v. The COR is the ratio v/v_o. In this model, the bat has only a single degree of
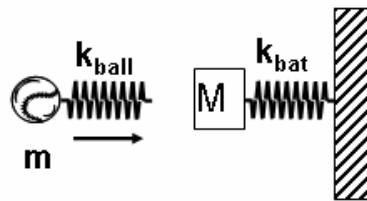
freedom corresponding to the vibration of M on the bat spring. All other degrees of freedom, such as the rigid body motion and other vibrational modes of the bat, are ignored; while they may be important to the understanding of how a real bat works, they are not essential to our understanding of the trampoline effect. This model is nearly identical to that used earlier (Nathan, 2000) to characterize the bending vibrations of the bat, except that in the latter case there was no dissipation in the ball.

The three parameters describing the ball (m, k_{ball}, and damping constant) are chosen to reproduce the known mass (5.2 oz), ball-wall collision time (~0.6 ms), and e_0 (0.5) for a baseball (Adair, 2002). The dependence of the COR on the two bat parameters M and k_{bat} is investigated. Actually it is more physically meaningful to investigate the dependence on two related factors. One of these is the ratio of spring constants $r_k=k_{bat}/k_{ball}$; the other is the product $f\tau$, where $2\pi f = \sqrt{k_{bat}/M}$ is the natural vibrational frequency of the bat spring and τ is the collision time, the latter determined primarily by the ball parameters. For $f\tau>1$, r_k is roughly proportional to the ratio of initial energy stored in the compression of the ball to that stored in the compression of the bat; therefore it is expected to play a crucial role in the trampoline effect. On the other hand, $f\tau$ determines the fraction of initial energy that is transferred to the vibrational mode of the bat, as will be seen by the explicit calculations described below. The effective bat mass M is initially set to be four times the ball mass (20.8 oz), and the dependence of the COR on r_k is investigated. The results are summarized in Fig. 2, which shows both the COR and how the final energy is partitioned.

It is both interesting and instructive to discuss initially the two limiting cases (Nathan, 2000). For $r_k>>1$ and $f\tau>1$, the bat looks completely rigid and therefore infinitely massive during the collision, so that the ball bounces from it as it would bounce from any massive rigid object; i.e., with $e=e_0$, with 25% of the initial energy going to the rebounding ball, with 75% of the initial energy dissipated in the ball, and with no energy transferred to the bat. This regime is denoted as the "strong coupling" limit, because the mass M is effectively rigidly attached to the wall. For $r_k<<1$ and $f\tau<<1$, the bat mass is completely decoupled from the spring on the short time scale of the collision. In effect, the ball bounces from the
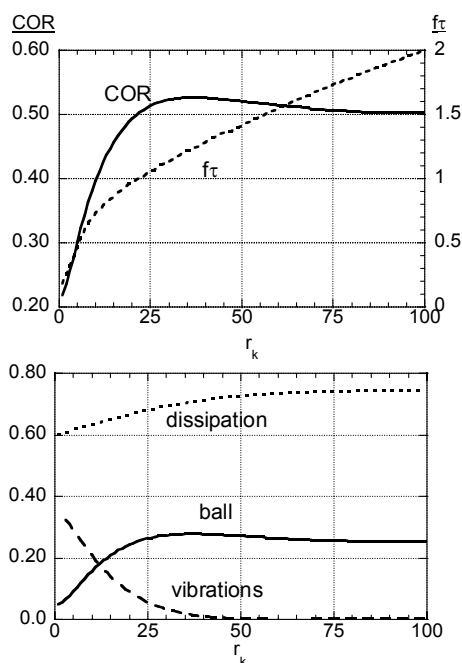

Figure 2. *COR and $f\tau$ (top) and energy fraction (bottom) as a function of r_k for M=20.8 oz.*

mass so quickly compared with the vibrational period of the bat that the mass has no time to recoil during the collision, so the ball bounces from an essentially free object of mass M. This interpretation is in complete accord with the numerical results for the COR as well as the partitioning of the initial energy into kinetic energy of the rebounding ball, energy dissipated in the ball, and energy transferred to the bat, the latter in the form of vibrations. This regime is denoted as the "quasi-free" limit, because the bat mass is essentially free during the collision. Note that for the particular mass chosen, the COR in the quasi-free limit is actually less than e_0, a fact that makes sense physically: a ball rebounds from a finite-mass object with less speed than from an infinite-mass object.

Starting from the strong coupling limit, as r_k is reduced with M fixed, more energy gets stored in the bat spring, less energy gets stored (and dissipated) in the ball spring, and the COR rises; for the rather large value of M chosen here, the rise is only modest and not typical of aluminum bats. The energy plots are consistent with this picture as they demonstrate that as r_k is reduced, the kinetic energy fraction of the rebounding ball increases, and the fraction of energy dissipated in the ball decreases. As long as the collision time is longer than $\sim 1/f$, no net energy is transferred to the bat. The physical picture is that the ball adiabatically pushes on the bat spring, initially compressing it, then releasing it, on a time scale long compared with the vibrational period of the spring. Under such conditions, all of the compressional energy of the bat is returned to the ball and none remains in the bat. With further reduction in r_k, and a consequent reduction in $f\tau$, a growing fraction of the energy stored in the bat remains in the bat after the collision. As a result, the COR curve does not continue to grow to unity, as it would in the absence of energy transferred to the bat. Instead it reaches a peak, then subsequently starts to fall because the energy transferred to the bat grows rapidly as $f\tau <<1$. These features are in accord with the curves in Fig. 2. Another way to look at the interplay between stored and transferred energy is shown in Fig. 3, which shows the time evolution of the ball and bat energies as well as the ball compressional energy for the fixed value $r_k=25$ corresponding to $f\tau \cong 1$. In this example, about 10% of the initial energy goes into compressing the bat and half of this is transferred to the bat. The value of r_k at which the COR is maximized occurs when $f\tau \cong 1$; therefore, for a smaller mass M, that value gets smaller and the peak value of the COR gets larger, as demonstrated in Fig. 4. Note that a larger fraction of the stored energy remains in the bat after the collision for the larger mass, which has a smaller value of $f\tau$. The ideal bat would have both $r_k <<1$, so that very little energy is stored and dissipated in the ball, and $f\tau >>1$, so that no energy is transferred to the bat. This could be achieved by a bat
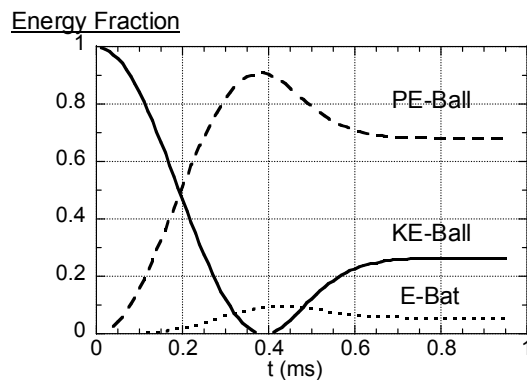

Figure 3. *Energy fraction versus time for $r_k=10$ and $M=20.8$ oz.*

consisting of a thin, deformable, but nearly massless membrane, for which the COR is nearly unity independent of e_0. Indeed, a completely dead ball with $e_0=0$ would be indistinguishable from a superball with $e_0\cong1$ when bounced from such a membrane (Adair, 2002).

Some practical consequences follow from our simple picture:

Figure 4. *COR vs .r_k for M=5 (solid), 10 (dotted), 15 (dashed), and 20 (dash-dotted) oz. The anomalous behavior for M=10 arises because of multiple collisions.*

- While on the rising part of the COR-vs.-r_k curve, the COR can be increased with either a softer bat (k_{bat} smaller) or a harder ball (k_{ball} larger). The effect of softball compression on bat performance is discussed at this conference (Duris and Smith, 2004).
- The ratio e/e_0, often referred to as the Bat Performance Factor or BPF (Nathan, 2003), is not independent of e_0 but rather decreases with increasing e_0.. Indeed the above example of the superball and dead ball is a dramatic, albeit extreme, demonstration of this point: The BPF is 1 for the superball and infinite for the dead ball. This conclusion is contrary to the commonly held belief that the BPF normalizes out the effect of the ball COR and so represents a bat property that is independent of e_0. Our conclusion is in accord with recent unpublished data taken at the Sports Science Laboratory (SSL) at Washington State University.
- A baseball is almost surely a nonlinear spring (Adair, 2002), with the effective spring constant increasing and e_0 decreasing with incident speed. As a consequence, the BPF grows with higher incident speed, in agreement with data taken at the SSL.
- For $r_k >>1$, the collision time depends only on the ball mass-spring system. However, as r_k is reduced from this limit, the effective spring constant is reduced as the bat spring also compresses, resulting in a somewhat longer collision time. Therefore a weak correlation is expected between e and collision time, a feature that is exploited in the pendulum test that is used to characterize the performance of golf drivers (USGA, 2003).
- For given ball parameters, the COR depends on both vibration parameters, *M* and k_{bat}. Because each parameter can be independently adjusted, the COR is not expected to be a unique function of the trampoline frequency alone. This issue is addressed in a contribution to these proceedings (Russell, 2004).

BEYOND THE SIMPLE PICTURE

An improvement to the simple picture is sought, using as a starting point the model of the ball-bat collision that was previously developed to treat the bending modes of solid wood bats (Nathan, 2000). For hollow bats, there are additional "hoop" modes due to
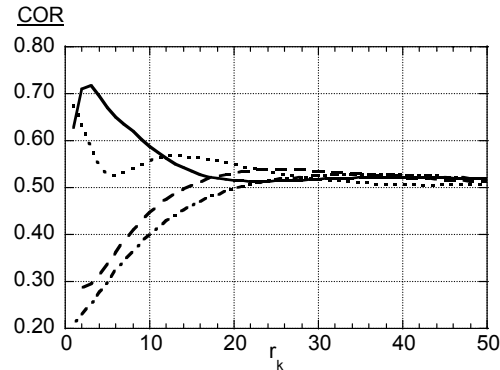
the deformation of the thin shell. The lowest hoop mode is a quadrupole-type deformation with 4 nodes in the azimuthal direction and a frequency typically in the range 1000-3000 Hz (Russell, 2004). From thin-shell theory, the stiffness of a shell for such a deformation scales with $(t/R)^3$, where t and R are the shell thickness and radius, respectively. Therefore, the mode is expected to have appreciable amplitude only in the fat part of the bat, in agreement with recent experiments (Russell, 2004). The previous model is modified by adding a single hoop
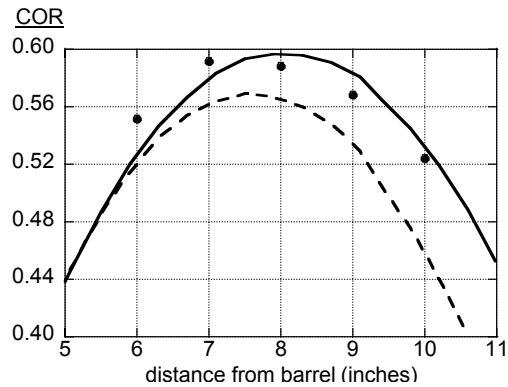

Figure 5. *Calculated (curves) and measured (points) COR vs. impact location.*

mode, whose properties (mass, modal shape, and spring constant) are derived from experimental data rather than from a first-principles calculation. This revised model is then used to study the collision between a softball and a particular softball bat, whose modal and collision properties have been studied experimentally. The resulting COR is plotted as a function of impact location in Fig. 5, along with experimental data taken at the SSL. The solid curve, representing the full calculation, faithfully accounts for the overall size and the spatial dependence of the COR, although a relative shift by ~1/2" would improve the agreement. The dashed curve is a calculation that eliminates the third bending mode, which has a frequency of 1174 Hz. Interestingly, eliminating this mode *decreases* the COR, suggesting that this mode contributes to the trampoline effect. The frequency of the mode is nearly optimum, given the collision time of approximately 1 ms. Lower frequency modes are net dissipaters of energy whereas higher frequency modes are too stiff to contribute to the trampoline effect. Closer inspection shows that this mode has an antinode 10" from the barrel end, close to the "sweet spot zone," defined as the region of the maximum of the COR curve. These observations suggest that improved performance of this bat might be obtained by redistributing the mass in order to move the antinode of the third bending mode closer to the sweet spot zone

DOES A CORKED BAT HAVE A TRAMPOLINE EFFECT?

A corked bat is a wood bat in which a cylindrical cavity is drilled axially into the barrel of the bat. Typically the diameter of the cavity is ~1" and the length ~l0". Often the cavity is filled with a light inert material, such as cork—hence, corking. By removing weight from the barrel region, the batter can achieve a higher swing speed and better bat control. This increased swing speed is at least partially compensated by a less effective collision due to the lower barrel weight. Some batters claim that the empty cavity gives rise to a trampoline effect, much like in hollow metal bats. Given the steep dependence of the hoop spring constant on wall thickness, this claim seems highly unlikely. For example, the thickness of a typical aluminum bat is only about 0.1"

whereas the wall thickness of a hollowed-out wood bat is perhaps 7 times larger. In order to investigate this question experimentally, the SSL facility was used to fire a baseball at 110 mph into the barrel of a bat that was initially at rest but free to pivot about the handle. The incoming and outgoing speeds of the ball were measured which were used along with kinematic formulas (Nathan, 2003) to determine the COR. A single baseball and standard wood bat were used. Initially the unmodified bat was impacted. Then a cavity was bored into the bat and the hollowed-out bat was impacted. Finally, the cavity was filled with cork and the corked bat was impacted. A "monitor bat" was used throughout the experiment to verify that the properties of the baseball had not changed as the result of repeated impacts. The results, presented in Fig. 6, show no appreciable difference among the three bats. It is concluded that there is no measurable trampoline effect from a hollowed or corked bat.
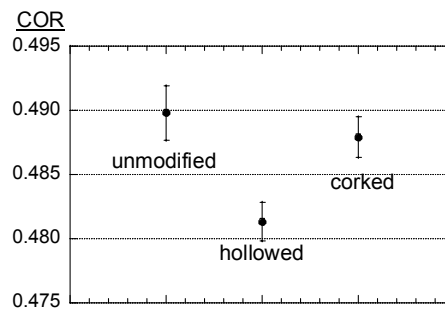

Figure 6. *Measured COR for an unmodified, hollowed, and corked wood bat.*

SUMMARY

The physics of the trampoline effect in baseball and softball bats has been investigated in the context of simple models of the ball-bat collision. The model provides excellent agreement with a variety of qualitatitve phenomena related to the performance of bats. No experimental evidence is found for a trampoline effect in hollowed or corked wood bats.

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