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IMPLEMENTATION OF DISCRETE FUZZY STRUCTURE MODELS IN *MATHEMATICA*

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SUMMARY

Since Soize introduced the concept of fuzzy structures in structural acoustics there has been little activity clarifying the basic elements which underlie his theory. Soize's papers are not easy reading due to the high level of mathematical formalism. In addition Soize simultaneously bases this fuzzy structure theory on two components: (1) a model for one Degree Of Freedom (DOF) fuzzy oscillators, and (2) a medium frequency solution method developed previously. It is unclear as to the role of the two components, although others have already undertaken a study of the medium frequency method by itself.

In the present paper a fundamental analysis of the first component, the one-DOF fuzzy oscillators, is undertaken. The symbolic manipulation program *Mathematica* is utilized to gain insight into this component of Soize's fuzzy theory. The resulting *Mathematica* simulations are easy to use and interpret, and they provide valuable insight into the parameters composing Soize's fuzzy oscillators. It is determined that in many cases of structural acoustics, where there is small damping and a medium to high modal density, the fuzzy mass primarily determines what effect a discrete fuzzy oscillator will have as an attachment.

INTRODUCTION

In a series of articles Dr. Christian Soize of Office National de Recherches d'Etudes Aerospatiale (ONERA) in Chatillon, France, has introduced the concept of fuzzy structures in the acoustic scattering from underwater structures.[1,2] In Soize's concept, that part of an underwater structure which is well known is called the master structure, and that part which is known imprecisely is called the structural fuzzy. See Figure 1, for example. To characterize the structural fuzzy, Soize represents the fuzzy as a collection of independent one Degree Of Freedom (DOF) oscillators with special properties. If one wanted to represent a finite circular cylindrical shell with hemispherical end caps as the master structure, one could represent the structural fuzzy by either (a) attaching discrete fuzzy one-DOF oscillators at specific points on the interior of the shell or (b) by smearing a distribution of fuzzy one-DOF oscillators per unit area over parts of the shell's internal surface.

In Soize's articles the fuzzy structure theory is outlined in combination with a method he has previously used for solving large finite element vibration simulations in the medium frequency range.[3] This medium frequency range method has been studied by others, independently of the fuzzy structure theory.[4] However, a lucid explanation of the fuzzy structure theory is lacking.

To fully understand the underlying basis for Soize's theory, the present authors have undertaken a study of Soize's special one-DOF oscillators. The results of this study are presented in this paper. It is found that Soize's fuzzy mass parameter primarily determines the effect of the

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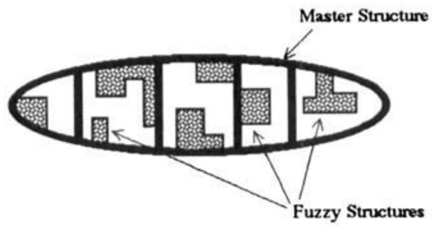

Figure 1. Soize's concept of a well-known master structure with attached internal structural fuzzies, known imprecisely

structural fuzzy on the master structure. Other effects due to Soize's fuzzy critical damping parameter and fuzzy modal density parameter are also explained.

These results were derived by modelling the one-DOF fuzzy oscillators using the symbolic manipulation program, *Mathematica*.[5] In fact, because of *Mathematica*'s programmability, availability of special functions, and easy to use notebook interface, the fuzzy simulations were implemented in a style not dissimilar from a commercial spreadsheet program. Outline of the rest of this paper is as follows: First, the underlying basis of Soize's fuzzy one-DOF oscillators is examined. Next, the implementation of the models in *Mathematica* is explained. The three parameters of the fuzzy oscillators are varied to form a large battery of simulations. These simulations are described next, and the results are provided. This paper then ends with some conclusions.

SOIZE'S FUZZY ONE-DOF OSCILLATORS

In the fuzzy structure theory Soize envisions that the structural fuzzy is implemented as attachments on the inside of the master structure. Although Soize allows for the fuzzy to be spread over finite patches of the master structure interior, as a fuzzy per unit area, the present research only addresses the case of discrete one-DOF fuzzy oscillators. The discrete oscillators will have a known attachment point. This is consistent with the original Soize theory, although recently Soize has introduced spatial-memory fuzzies which do not have this restriction.[6] As mentioned previously, when Soize actually implements a fuzzy structural dynamics simulation, a medium frequency method is also employed.[3] This medium frequency technique is not used in the present paper so as to clarify the role of the fuzzy oscillators themselves, apart from the medium frequency technique.

The discrete oscillators examined here are assumed to be excited only in the direction normal to the surface at which they are attached. A normal displacement $U(\omega)$ is generated due to the master structure moving and exerting a force $F(\omega)$ on the fuzzy. In Figure 2 one can see that a one-DOF system is composed of a fuzzy mass μ, a fuzzy stiffness K and a fuzzy damping C. However, instead of using these parameters directly, Soize prefers to use the parameters μ for fuzzy mass, ω_p for fuzzy natural frequency, and ξ for a fuzzy critical damping ratio. The two new quantities are related to the others by

$$\omega_p = (K/\mu)^{1/2} \tag{1}$$

and

$$\xi = \frac{C}{2\mu\omega_p} \tag{2}$$

Here, the discrete oscillators are assumed to be weakly damped so $0 < \xi < 1$.

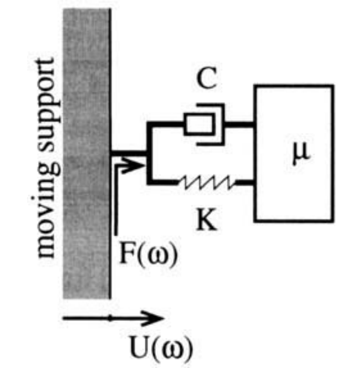

Figure 2. A discrete fuzzy one-DOF oscillator attached to the master structure. The master structure moves with normal displacement $U(\omega)$ and exerts a force $F(\omega)$ on the oscillator. The variables μ, K and C represent mass, stiffness and damping, respectively

In Soize's original detailed analysis, a fuzzy one-DOF oscillator was also represented by a simpler model for low frequency, below some cutoff frequency Ω_c. This simpler model assumes no stiffness and damping but only mass, since for low frequency this is all the master structure 'sees' as an impedance looking at the fuzzy. Because this special case is contained in the more general one, the one-DOF model including all three parameters: mass, stiffness and damping, will be considered in the present analysis. This is equivalent to setting $\Omega_c = 0$ in Soize's formulation.

Since Soize wishes for the one-DOF model to be as general as possible, he considers the fuzzy mass, resonant frequency and critical damping ratio to be functions of frequency as $\mu(\omega)$, $\omega_p(\omega)$ and $\xi(\omega)$. Further, since one may not know the specific resonant frequency of the fuzzy oscillator, Soize represents this parameter $\omega_p(\omega)$ indirectly, by way of a fuzzy modal density $n(\omega)$ with units s/rad. The fuzzy modal density $n(\omega)$ is the number of natural frequencies that the fuzzy has within one rad/s. The use of a fuzzy modal density allows for the one-DOF oscillator to have an uncertain natural frequency.

The three parameters that Soize employs in his simulations, and are under study in the present context, are $\mu(\omega)$, $\xi(\omega)$ and $n(\omega)$. To make some assumption about these parameters, Soize assumes that they take on some mean values $\underline{\mu}(\omega)$, $\underline{\xi}(\omega)$ and $\underline{n}(\omega)$, and have some deviation about the mean values in the following way:

$$\mu(\omega) = \underline{\mu}(\omega)[1 + Y_1] \tag{3}$$

$$\xi(\omega) = \underline{\xi}(\omega)[1 + Y_2] \tag{4}$$

$$n(\omega) = \underline{n}(\omega)[1 + Y_3] \tag{5}$$

Here Y_1, Y_2 and Y_3 are random variables, taking on values which have absolute values assumed small compared to 1.

In the simulations the Y_i, for i = 1, 2, 3, are defined by dispersion parameters λ_i and by random variables X_i. The variables X_i are real, mutually independent random variables which have a uniform probability density over the interval $[-\sqrt{3}, \sqrt{3}]$. The Y_i are given by

$$Y_i = X_i \frac{\lambda_i}{\sqrt{3}} \tag{6}$$

For example, suppose $\lambda_1 = 0{\cdot}2$, $\lambda_2 = 0{\cdot}0$ and $\lambda_3 = 0{\cdot}1$. These values mean that the fuzzy mass will deviate about its mean by $\pm$ 20 per cent, the fuzzy critical damping ratio will not vary from its mean value, and the fuzzy modal density will deviate about its mean by $\pm$ 10 per cent. Further, the probability densities for the Y_i are written, assuming Y_i is known to have value y_i, as

$$p_i(\omega, y_i) = \frac{1}{2\lambda_i} \tag{7}$$

given that y_i is between $-\lambda_i$ and λ_i, and $p_i(\omega, y_i)$ is zero otherwise. Additional details are available in Reference 1.

To get back the natural frequency of the fuzzy $\omega_p(\omega)$ from the definition of the fuzzy modal density in equation (5), the following analysis is required, reproduced here from Reference 1. Define $2\varepsilon(\omega)$ to be distance between two natural frequencies in the vicinity of ω. Hence,

$$2\varepsilon(\omega) = \frac{1}{n(\omega)} \tag{8}$$

This means that, knowing $Y_3 = y_3$, which is equivalent to knowing $n(\omega)$, one can get the probability density of $\omega_p(\omega)$. Mathematically, the conditional probability density of ω_p given $Y_3 = y_3$ is

$$p_{\omega_p(\omega)}(\tilde{\omega}, \omega \,|\, y_3) = \frac{1}{2\varepsilon(\omega)} \tag{9}$$

for frequencies $\tilde{\omega}$ between $\omega - \varepsilon(\omega)$ and $\omega + \varepsilon(\omega)$ and is zero otherwise. Thus,

$$p_{\omega_p(\omega)}(\tilde{\omega}, \omega \,|\, y_3) = n(\omega) \tag{10}$$

for frequencies $\tilde{\omega}$ between $\omega - b(\omega, y_3)$ and $\omega + b(\omega, y_3)$ and is zero otherwise. Here the new function $b(\omega, y_3)$ is defined by

$$b(\omega, y_3) = \frac{1}{2\underline{n}(\omega)[1 + y_3]} \tag{11}$$

Equation (10) is a conditional probability density function for $\omega_p(\omega)$. To find the probability density for $\omega_p(\omega)$ itself, one can use

$$p_{\omega_p(\omega)}(\tilde{\omega}, \omega) = \int p_{\omega_p(\omega)}(\tilde{\omega}, \omega \,|\, y_3)\, p_3(\omega, y_3)\, \mathrm{d}y_3 \tag{12}$$

where $p_3(\omega, y_3)$ is the probability density for Y_3 defined in equation (7) and where the integration is taken over all possible values of y_3. Performing the integral in equation (12), Soize finds that

$$p_{\omega_p(\omega)}(\tilde{\omega}, \omega) = \begin{cases} \underline{n}(\omega), & \omega - b(\omega) \leqslant \tilde{\omega} \leqslant \omega + b(\omega) \\ \dfrac{1}{16\lambda_3 \underline{n}(\omega)(\omega - \tilde{\omega})^2} - \underline{n}(\omega)\dfrac{(\lambda_3 - 1)^2}{4\lambda_3}, & \omega - a(\omega) \leqslant \tilde{\omega} \leqslant \omega - b(\omega) \\ \dfrac{1}{16\lambda_3 \underline{n}(\omega)(\omega - \tilde{\omega})^2} - \underline{n}(\omega)\dfrac{(\lambda_3 - 1)^2}{4\lambda_3}, & \omega + b(\omega) \leqslant \tilde{\omega} \leqslant \omega + a(\omega) \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

where

$$a(\omega) = \frac{1}{2\underline{n}(\omega)(1 - \lambda_3)}$$

and

$$b(\omega) = \frac{1}{2\underline{n}(\omega)(1 + \lambda_3)}$$

This expression is what makes for some difficulty in simulating the fuzzy one-DOF oscillator in a computer program. Numerically integrating equation (13) is not straightforward.

One additional point must be made. In Soize's model the master structure sees the fuzzy as the quantity

$$Z(\omega) = \frac{F(\omega)}{U(\omega)} \tag{14}$$

where $F(\omega)$ is the excitation force of the structure on the fuzzy and $U(\omega)$ is the displacement of the master structure at this point. Here

$$Z(\omega) = -\omega^2 R(\omega) + \mathrm{i}\omega I(\omega) \tag{15}$$

where

$$R(\omega) = \frac{(\omega_p^2/\omega^2)[(\omega_p^2/\omega^2) - 1 + 4\xi^2]\mu}{[(\omega_p^2/\omega^2) - 1]^2 + 4(\omega_p^2/\omega^2)\xi^2} \tag{16}$$

and

$$I(\omega) = \frac{2\mu\omega_p\xi}{[(\omega_p^2/\omega^2) - 1]^2 + 4(\omega_p^2/\omega^2)\xi^2} \tag{17}$$

are defined in terms of the three fuzzy parameters $\mu(\omega)$, $\xi(\omega)$ and $\omega_p(\omega)$. Soize calls Z a boundary impedance in his original paper,[1] which is not correct. Note that this terminology is corrected in his later work[6] where he defines

$$Z(\omega) = \frac{F(\omega)}{\mathrm{i}\omega U(\omega)} \tag{18}$$

Soize consistently uses the $\mathrm{e}^{\mathrm{i}\omega t}$ time convention.

MODELLING THE DISCRETE FUZZY OSCILLATORS IN *MATHEMATICA*

Equation (13) gives the probability density function for the natural frequency of the fuzzy oscillator. Let us now define the cumulative probability distribution of the natural frequency of the fuzzy oscillator as:

$$P_{\omega_p(\omega)}(\omega_{\mathrm{upper}}, \omega) = \int_{-\infty}^{\omega_{\mathrm{upper}}} p_{\omega_p(\omega)}(\tilde{\omega}, \omega)\,\mathrm{d}\tilde{\omega} \tag{19}$$

where ω_{upper} is the upper limit on the integration. Clearly, from introductory probability theory, if $\omega_{\mathrm{upper}} \to \infty$ then $P_{\omega_p(\omega)} = 1$. Looking at equation (13) more carefully, however, it is apparent that one can write equation (19) as

$$P_{\omega_p(\omega)}(\omega_{\mathrm{upper}}, \omega) = \int_{\omega - a(\omega)}^{\omega_{\mathrm{upper}}} p_{\omega_p(\omega)}(\tilde{\omega}, \omega)\,\mathrm{d}\tilde{\omega} \tag{20}$$

and $P_{\omega_p(\omega)}$ will equal 1 when ω_{upper} equals $\omega + a(\omega)$.

This is the equation by which the natural frequencies of the fuzzy one-DOF oscillators are picked in the simulations. If one picks seeds from a uniform distribution of real numbers between

0 and 1 and then equates these seeds to $P_{\omega_p(\omega)}$, the corresponding values of ω_{upper} denote the natural frequencies of the fuzzy oscillators. In this formulation, one controls the probability density function $p_{\omega_p(\omega)}$ using the fuzzy modal density parameter $n(\omega)$, and through equation (20) $n(\omega)$ controls the selection of the natural frequencies.

These equations were coded into a notebook of the symbolic manipulation program *Mathematica*. Because equation (13) is not a smooth function, it is necessary to define the integral in equation (20) carefully, telling *Mathematica* where are the discontinuities in slope of equation (13). The *Mathematica* `NIntegrate` function is used to integrate equation (20).

Before any realizations of natural frequencies are computed in a simulation, a large look-up table is generated where each entry in the table contains (a) a value of ω_{upper} and (b) a corresponding value of $P_{\omega_p(\omega)}$. This step simply calls for integrating equation (20) as many times as is necessary to fill out the table, given some range of ω_{upper}.

Then to actually pick a natural frequency, one picks a real random number out of a uniform distribution from 0 to 1. The second column of the look-up table is searched to find the corresponding value of $P_{\omega_p(\omega)}$, and then the corresponding value of ω_{upper} is found. This value ω_{upper} is chosen as the natural frequency $\omega_p(\omega)$ of the fuzzy oscillator. This value can be inserted in equations (16) and (17) along with the values of $\mu(\omega)$ and $\xi(\omega)$.

The algorithm for the calculation of an impedance curve of a single discrete fuzzy oscillator, as implemented in *Mathematica*, is as follows:

(1) Set the dispersion parameters for the fuzzy, λ_1, λ_2 and λ_3.
(2) Set the mean values of mass, critical damping ratio and modal density, $\underline{\mu}$, $\underline{\xi}$ and $\underline{n}$. Also pick a mean value for the natural frequency of the fuzzy ω_p.
(3) Set up the random variables X_1, X_2 and X_3.
(4) Calculate $\mu(\omega)$, $\xi(\omega)$ and $n(\omega)$ from equations (3)–(5).
(5) Calculate the frequencies for which equation (13) has a discontinuity in slope, $\omega \pm a(\omega)$ and $\omega \pm b(\omega)$.
(6) Define equation (13).
(7) Create the look-up table by picking values of ω_{upper}, and integrating equation (20) for each.
(8) Define ω_p to be a function which searches through the look-up table, given a random number from a uniform distribution from 0 to 1.
(9) Define the function $Z(\omega)$ from equations (15)–(17).
(10) Plot the function $Z(\omega)/(i\omega)$ versus frequency for the one-DOF fuzzy oscillator. For each frequency the function is called, *Mathematica* will generate appropriate $\mu(\omega)$, $\xi(\omega)$ and $\omega_p(\omega)$ given their previous definitions. In the case of ω_p, the value is found from the look-up table, based on the previously given definition of $n(\omega)$.

This mode of using *Mathematica* is called *rule based programming. Mathematica* acts like an expert system by storing specified sets of rules concerning the variables for later use. When the function is actually plotted, the rules are used to find appropriate values for the function.

This method of calculation is not dissimilar from how one uses a traditional spreadsheet program. The difference between *Mathematica* and a standard spreadsheet program is that *Mathematica* has the ability to perform certain high-level mathematical operations that spreadsheets cannot.

It is important to note that for each frequency in a particular plot a new $\mu(\omega)$, $\xi(\omega)$ and $\omega_p(\omega)$ is accessed. The plots of $Z(\omega)/i\omega$, therefore, will not be smooth functions. Instead, they will be scatter plots showing the general character of a number of plots in which $\mu(\omega)$, $\xi(\omega)$ and $\omega_p(\omega)$ would have been fixed as functions of frequency.

Suppose one gives the command in step number 10 above repeatedly. Because of the probabilistic definition of the rules, every scatter plot that *Mathematica* makes for $Z(\omega)/(i\omega)$ will be different, although the λ_i's and μ, ξ and $\underset{\sim}{n}$ remain the same. However, each plot will have a statistically similar distribution of data points and have the same general shape.

SIMULATIONS AND RESULTS

Numerous runs were made of the *Mathematica* notebook 'spreadsheet'. In this section some initial definitions are given, and then a selection of results are presented.

In the simulations the mean of the fuzzy natural frequency ω_p was always picked to be 50 rad/s. If no uncertainty in natural frequency was desired, a large value (≈ 100) of $\underset{\sim}{n}$ was used, along with $\lambda_3 = 0$. This ensures that a natural frequency occurs very close to 50 rad/s. If a wider variation in natural frequency was desired, a much smaller ($\approx 0{\cdot}1$) value of $\underset{\sim}{n}$ was used. The value $\underset{\sim}{n}$ equal to 0·1 with $\lambda_3 = 0$ indicates that there is approximately a 1-in-10 chance that a natural frequency will appear in the frequency range of 49·5 to 50·5 rad/s. For a non-zero λ_3, the distribution of the natural frequencies is allowed to vary above or below the distribution corresponding to $\lambda_3 = 0$.

As a baseline for comparison a simulation was run with $\lambda_1 = \lambda_2 = \lambda_3 = 0$, $\underset{\sim}{\mu} = 1$ kg, $\underset{\sim}{\xi} = 0{\cdot}1$ and $\underset{\sim}{n} = 100$ s/rad. The result is seen in Figure 3 and clearly is the impedance curve of a one-DOF oscillator with no fuzziness. Here the magnitude of the force-to-velocity ratio, equal to $Z(\omega)$ from equation (15) divided by $i\omega$, is plotted versus the driving frequency. If Figure 3 looks odd to the reader, recall that $Z(\omega)/(i\omega)$ represents the ratio of force that the master structure is exerting on the oscillator to the normal velocity of the master structure. This is a different convention for considering a one-DOF oscillator from that given in elementary mechanics textbooks, where the master structure is considered rigid, and where a force is applied to the mass and the velocity of the mass is examined.

If some uncertainty in the mass is allowed, say up to 20 per cent variation, then λ_1's value is changed to 0·20. The result is plotted, for a typical plot, as Figure 4. The impedance curve now has some fuzziness, and by overlaying Figures 3 and 4 one can see that Figure 4 has some deviation above and below the values on Figure 3, the non-fuzzy result. One should recall that Figure 4 is a typical plot and that rerunning the simulation with $\lambda_1 = 0{\cdot}2$ will result in a plot with similar spread to Figure 3, but with different specific values for each frequency ω.

A large matrix of runs was made varying all of the relevant parameters of the fuzzy model. Some example runs are now described. First taking the parameters for Figure 3 for the baseline non-fuzzy result, if the critical damping ratio dispersion parameter λ_2 is changed from 0 to 0·2, with $\lambda_1 = \lambda_3 = 0$, Figure 5 results. In this case it is clear that fuzziness in the damping is primarily

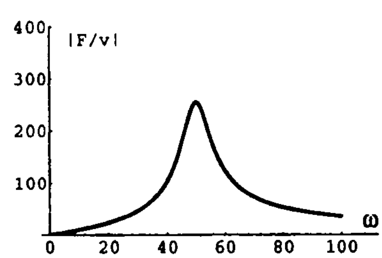

Figure 3. Magnitude of impedance (N s/m) versus frequency (rad/s) curve for a one-DOF fuzzy oscillator with no uncertainty. Here $\lambda_1 = \lambda_2 = \lambda_3 = 0$, $\underset{\sim}{\mu}(\omega) = 1{\cdot}0$ kg, $\underset{\sim}{\xi}(\omega) = 0{\cdot}1$ and $\underset{\sim}{n}(\omega) = 100$ s/rad

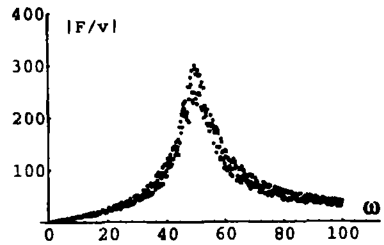

Figure 4. Magnitude of impedance (N s/m) versus frequency (rad/s) curve for a one degree of freedom fuzzy oscillator with fuzzy mass. Here $\lambda_1 = 0{\cdot}2$, $\lambda_2 = \lambda_3 = 0$, $\underline{\mu}(\omega) = 1{\cdot}0$ kg, $\underline{\xi}(\omega) = 0{\cdot}1$ and $\underline{n}(\omega) = 100$ s/rad

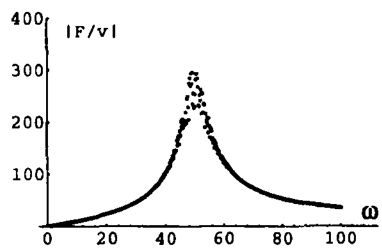

Figure 5. Magnitude of impedance (N s/m) versus frequency (rad/s) curve for a one degree of freedom fuzzy oscillator with fuzzy damping. Here $\lambda_1 = 0$, $\lambda_2 = 0{\cdot}2$, $\lambda_3 = 0$, $\underline{\mu}(\omega) = 1{\cdot}0$ kg, $\underline{\xi}(\omega) = 0{\cdot}1$ and $\underline{n}(\omega) = 100$ s/rad

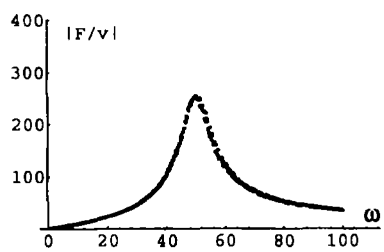

Figure 6. Magnitude of impedance (N s/m) versus frequency (rad/s) curve for a one degree of freedom fuzzy oscillator with fuzzy modal density. Here $\lambda_1 = \lambda_2 = 0$, $\lambda_3 = 0{\cdot}2$, $\underline{\mu}(\omega) = 1{\cdot}0$ kg, $\underline{\xi}(\omega) = 0{\cdot}1$ and $\underline{n}(\omega) = 1$ s/rad

manifested for driving frequencies near the natural frequency of the oscillator. This result is intuitive.

Further, if the baseline parameters are modified such that $\underline{n} = 1$ s/rad and $\lambda_3 = 0{\cdot}2$ with all other values unchanged, the impedance function takes the form seen in Figure 6. This shows little deviation from Figure 3. If the value of $\underline{n}$ is further decreased to 0·1, a large deviation in form is seen in Figure 7. It is easily seen here that with the small mean modal density that uncertainty has been added in the natural frequency of the fuzzy. One can interpret this uncertainty in the impedance curve by imagining that the impedance curve of Figure 3 has been shifted from side to side in a random way.

The major results of the large matrix of runs are as follows:

- For many situations which arise in structural acoustics, for which the damping is weak, and for moderate-to-high modal densities, the mass is the primary factor determining the effect of the

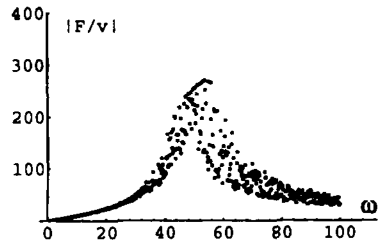

Figure 7. Magnitude of impedance (N s/m) versus frequency (rad/s) curve for a one degree of freedom fuzzy oscillator with a smaller fuzzy modal density. Here $\lambda_1 = \lambda_2 = 0$, $\lambda_3 = 0{\cdot}2$, $\underline{\mu}(\omega) = 1{\cdot}0$ kg, $\underline{\xi}(\omega) = 0{\cdot}1$ and $\underline{n}(\omega) = 0{\cdot}1$ s/rad

fuzzy on the master structure. This assumes that the mean natural frequency of the oscillator under examination has a fixed value.

- The magnitude of the impedance curve is proportional to the mean mass of the fuzzy, $\underline{\mu}$. Increased uncertainty in the mass results in larger deviations in the magnitude of the curve above and below the mean mass value.
- When the mean critical damping ratio is small ($\approx 0{\cdot}1$) fuzziness in the damping is primarily seen near the oscillator's natural frequency. Higher uncertainty in the damping ratio corresponds to greater variations in the curve magnitude near the natural frequency.
- When the mean critical damping is large ($\approx 0{\cdot}5$) the magnitude of the impedance curve is decreased everywhere, and the curve is smoother.
- A moderate or high modal density affects the impedance curve very little. Values of $\underline{n} = 100$ and $\underline{n} = 1{\cdot}0$ s/rad give results which are not strikingly different.
- A low value of modal density has a profound effect on the impedance curve, spreading it out from side to side, but barely affecting the magnitude of the impedance.

CONCLUSIONS

This paper has introduced a method for modelling the fuzzy one-DOF oscillators which form the basis of the fuzzy structure theory of Soize. The fuzzy oscillators are examined apart from the medium frequency method Soize uses in his fuzzy simulations. The discrete fuzzy oscillator model described in this paper is straightforward, and one can perform repeated simulations by using the symbolic manipulation program *Mathematica* as one uses a spreadsheet program. The roles of various parameters in Soize's discrete fuzzy models were determined after performing a large matrix of runs, changing values in the *Mathematica* simulations. It was determined that if the damping in the fuzzy is assumed small and if the modal density of the combined fuzzy structure/master structure is moderate or large (which is often the case in structural acoustics problems), the fuzzy mass primarily determines the effect of the oscillator on the master structure. Future simplified models of fuzzy structures, therefore, may well be able to model complicated internals using only distributed mass–spring systems, neglecting the uncertainty in modal density and in critical damping ratio.

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