Acoustic monopoles, dipoles, and quadrupoles: An experiment revisited

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A simple and inexpensive demonstration of acoustic monopole, dipole, and quadrupole sources utilizes four 4-in. boxed loudspeakers and a homemade switch box. The switch box allows the speakers to be driven in any combination of phase relationships. Placing the speakers on a rotating stool allows students to measure directivity patterns for monopole, dipole, and quadrupole speaker combinations. Stacking the speakers in a square, all facing the same direction, allows students to aurally compare the frequency and amplitude dependence of sound radiation from monopoles, dipoles, and quadrupoles.

I. INTRODUCTION

Directivity patterns representing the angular distribution of the sound field radiated by acoustic monopole, dipole, and quadrupole sources are not usually topics covered in a typical undergraduate physics curriculum. However, the simple and inexpensive experiment discussed in this paper can effectively introduce students to a basic understanding of fields and sources, as well as interference effects, phase relationships, and polar plots. In addition, the direct analogy between acoustic[1,2] and electromagnetic[3,4] monopoles, dipoles, quadrupoles, and multipole expansions may benefit physics students taking a junior- or senior-level electromagnetic fields course.

Meyer and Neumann[5] describe a simple, but effective experiment to observe acoustic monopoles, dipoles, and quadrupoles using a combination of boxed and unbaffled loudspeakers. They show that a small boxed loudspeaker at low frequencies acts as a simple source, while an unboxed loudspeaker at low frequencies acts as a dipole source. They also show some experimental measurements of directivity patterns for monopole, dipole, and quadrupole sources. In addition they describe a simple demonstration of the frequency dependence of the sound power radiated by such sources.

In this paper we revisit this experiment with an apparatus of four identical boxed loudspeakers and a switch box which allows one to easily demonstrate or experiment with the directionality and frequency characteristics of monopole, dipole, lateral quadrupole, and longitudinal quadrupole sources. This apparatus is used by students for an experiment in a senior-level course in acoustics at Kettering University and provides a background for further experiments involving more complicated directivity patterns from baffled loudspeakers[6] and noise control.

II. MONOPOLES, DIPOLES, AND QUADRUPOLES

The directivity of a sound source refers to the manner in which the measured or predicted sound pressure, at a fixed distance r from the source, varies with angular position θ. For all plots in this paper, sound pressures are converted to sound pressure levels,

$$L_p(r,\theta)=10\log\frac{\langle|p^2(r,\theta,t)|\rangle_t}{p_{\mathrm{ref}}^2}, \quad (1)$$

where $\langle\,\rangle_t$ indicates a time average at a fixed (r,θ), $p_{\mathrm{ref}}=20\,\mu\mathrm{Pa}$ is the standard reference pressure, and the units of L_p, are decibels (dB). In addition, all sound pressure level values have been normalized to the value at $\theta=0°$ as is the accepted practice for directivity plots.[7]

An acoustic monopole radiates sound equally in all directions. An example of an acoustic monopole would be a small sphere whose radius alternately expands and contracts. In practice, any sound source whose dimensions are much smaller than the wavelength of the sound being radiated will act as a monopole, radiating sound equally well in all directions. This relationship between wavelength and dimension for a monopole is usually expressed as $ka\ll1$, where $k=2\pi/\lambda$ is the wave number, λ is the wavelength, and a is a characteristic dimension of the source.

The far field of an acoustic source is the sound field at a distance r from the source such that $kr\gg1$. The far-field pressure radiated by a monopole may be written as[8]

$$p(r,\theta,t)=i\,\frac{Q\rho ck}{4\pi r}\,e^{i(\omega t-kr)}, \quad (2)$$

and the pressure amplitude is then

$$|p(r,\theta,t)|=\frac{Q\rho ck}{4\pi r}, \quad (3)$$

where ρ is the fluid density, c is the speed of sound, k is the wave number, and r is the distance from source to observation point. Q is a constant, termed the *complex source strength* and represents the volume of fluid displaced by the source at the rate[8]

$$\mathbf{Q}e^{j\omega t}=\int\int\vec{\mathbf{u}}\cdot\hat{n}\,dS,$$

where $\vec{\mathbf{u}}$ is the velocity at some point on the surface of the source. For a pulsating sphere the source strength is real, and equals the product of surface area and surface velocity: $Q=4\pi a^2U_0$. The pressure amplitude in Eq. (3) does not depend on angle; the pressure produced by a monopole is the same at all points a distance r from the source. Thus the directivity pattern looks like a circle as shown in Fig. 1(a).

The sound power radiated by a monopole source is given by[9]

$$\Pi=\frac{Q^2\rho ck^2}{8\pi}\Rightarrow\Pi\sim\omega^2. \quad (4)$$

Since $k=\omega/c$ this means that the sound power radiated by a monopole varies as the square of frequency (for a fixed value of Q).

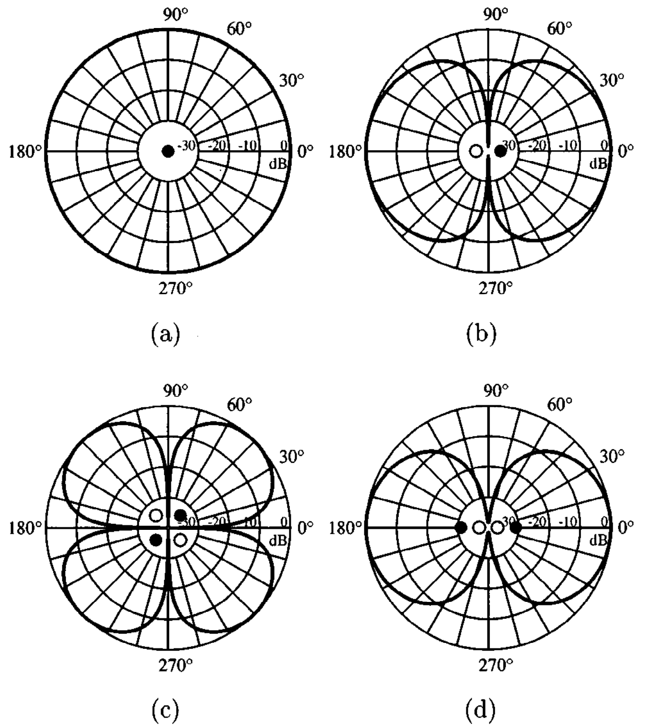

Fig. 1. Theoretical directivity patterns for far-field sound pressure levels radiated from (a) monopole, (b) dipole, (c) lateral quadrupole, and (d) longitudinal quadrupole sound sources.

Two monopoles of equal source strength, but opposite phase, and separated by a small distance d (such that $kd \ll 1$) comprise an acoustic *dipole*. In contrast to a single monopole, there is no net introduction of fluid by a dipole. As one source ''exhales,'' the other source ''inhales'' and the fluid surrounding the dipole simply sloshes back and forth between the sources. It is the net force on the fluid which causes energy to be radiated in the form of sound waves.

The far-field expression for the pressure radiated by an acoustic dipole may be written as[10]

$$p(r,\theta,t) = -i\,\frac{Q\rho c k^2 d}{4\pi r}\cos\theta e^{i(\omega t - kr)}. \tag{5}$$

This is a spherically diverging wave with pressure amplitude

$$|p(r,\theta,t)| = \underbrace{\frac{Q\rho c k}{4\pi r}}_{\text{simple source}}\; kd \underbrace{\cos\theta}_{\text{directivity}}\,, \tag{6}$$

which may be interpreted as the product of the pressure amplitude radiated by a monopole, a term kd which relates the radiated wavelength to the source separation, and a directivity function which depends on the angle θ. A dipole does not radiate equally in all directions. Instead its directivity pattern, as shown in Fig. 1(b), has maxima along the 0° and 180° directions, and no sound radiation along the 90° and 270° directions.

The sound power radiated by an acoustic dipole may be expressed as[9]

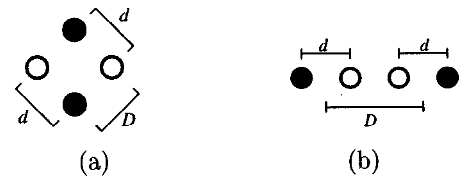

Fig. 2. (a) Lateral quadrupole and (b) longitudinal quadrupole source.

$$\Pi_D = \frac{Q^2\rho c k^4 d^2}{6\pi} \Rightarrow \Pi \sim \omega^4. \tag{7}$$

The dipole power varies with frequency as ω^4, which means that a dipole is less efficient than a monopole (with the same source strength) at radiating low frequency sounds.

A quadrupole source consists of two identical dipoles, with opposite phase and separated by small distance D. In the case of the quadrupole, there is no net flux of fluid and no net force on the fluid. It is the fluctuating stress on the fluid that generates the sound waves. However, since fluids don't support shear stresses well, quadrupoles are poor radiators of sound. For a *lateral quadrupole* source the dipole axes do not lie along the same line, as shown in Fig. 2(a), while for a *longitudinal quadrupole* source, the dipole axes do lie along the same line, as shown in Fig. 2(b).

The far-field sound pressure amplitude produced by a lateral quadrupole may be written as[10]

$$|p(r,\theta,t)| = \underbrace{\frac{Q\rho c k}{4\pi r}}_{\text{simple source}}\; 4k^2 dD \underbrace{\cos\theta\sin\theta}_{\text{directivity}}, \tag{8}$$

which may be interpreted as the product of a simple source, a term $4k^2dD$ which relates the radiated wavelength to the quadrupole source separations, and a directivity function which depends on the angle θ. There are four directions where sound is radiated very well, and four directions in which destructive interference occurs and no sound is radiated.

The far-field sound pressure amplitude produced by a longitudinal quadrupole may be written as[10]

$$|p(r,\theta,t)| = \underbrace{\frac{Q\rho c k}{4\pi r}}_{\text{simple source}}\; 4k^2 dD \underbrace{\cos^2\theta}_{\text{directivity}}\,, \tag{9}$$

which can be interpreted as the product of a simple source, the dimensionless size term $4k^2dD$, and the directivity function which depends on the angles θ. This directivity pattern, shown in Fig. 1(d), looks similar to that of the dipole source in Fig. 1(b). There are two directions in which sound is radiated extremely well, and two directions in which no sound is radiated. However, the width of the lobes is narrower than for the dipole; at 60° the longitudinal quadrupole directivity is approximately 5 dB less than that of the dipole.

The power radiated by a quadrupole varies according to the sixth power of frequency ($\Pi_Q \sim \omega^6$), which means that quadrupoles should be even less efficient radiators of low frequencies than dipoles (with the same source strength).[9]

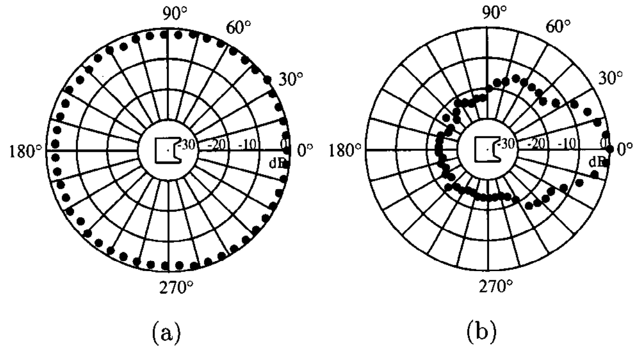

Fig. 3. Measured directivity patterns for a single boxed 4-in. speaker at (a) 250 Hz and (b) 10 kHz.

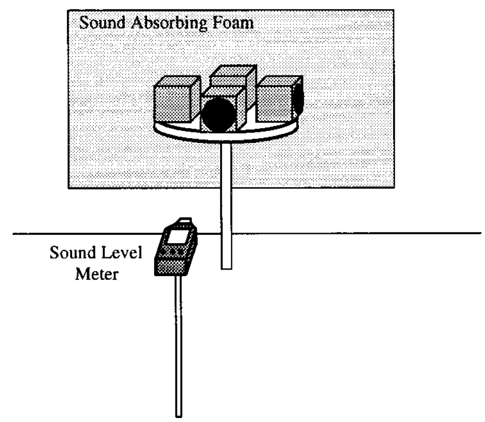

Fig. 5. Apparatus for demonstrating and measuring directivity patterns for monopoles, dipoles, and quadrupoles.

III. MEASURED DIRECTIVITY PATTERNS

In this experiment, our simple source is a 4-in. boxed loudspeaker[11] producing a 250-Hz pure sine tone. For this source, $a = 4\text{ in.} = 0.10\text{ m}$ and $k = 2\pi/\lambda = 2\pi f/c = 4.58$, where $c = 343\text{ m/s}$ is the speed of sound in air at room temperature (20 °C). Thus, for our source, $ka = 0.46$, which is close to the simple source approximation. To verify that this loudspeaker was indeed behaving as a simple source at 250 Hz, the speaker was placed on a rotating stool at a height of 80 cm above a carpeted floor. A sound level meter[12] was placed at the same height and 1 m from the speaker, pointed toward the speaker. Figure 3(a) shows the measured directivity pattern for a single speaker. At 250 Hz, it clearly behaves as a simple source, essentially radiating sound equally well in all directions. At higher frequencies, however, the speaker becomes very directional, as is shown in Fig. 3(b) for a 10-kHz pure sine signal. While the main emphasis of this experiment is to measure the directivity patterns of various sources, we did find that the loudspeakers used in this experiment have a relatively flat frequency response over the range 100 Hz–10 kHz.

In order to measure the directivity pattern of monopole, dipole, and quadrupole sources, four 4-in. boxed loudspeakers were symmetrically placed on a rotating stool, facing outwards, as shown in Fig. 4. Each speaker was wired to a double-pole-double-throw switch so that the speaker polarity, or phase, could be reversed by simply throwing the switch. The speakers were driven by an amplified sinusoidal signal; the single channel output of the amplifier was split four ways using *Y* connectors. To ensure that each of the four speakers were acting as identical sources, the sound pressure level directly in front of each speaker was measured; for our four speakers the results varied by less than 1 dB.

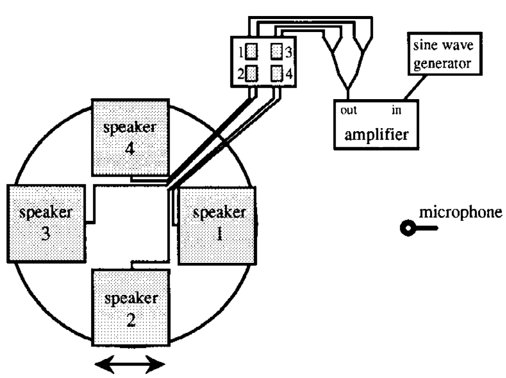

Fig. 4. Apparatus for demonstrating and measuring directivity patterns for monopoles, dipoles, and quadrupoles.

The directivity of the sound field produced by the speaker configuration was measured by a stationary sound level meter placed at a distance of 1 m from the center of the speaker arrangement as shown in Fig. 5. Alternately, the output from a cheap electret microphone could be displayed on an oscilloscope. The biggest difficulty we faced in collecting clean data was the elimination of reflections from nearby walls or obstacles and standing waves in the room in which the experiment was performed. The speakers were placed on a rotating stool 1 m off the floor, with the center of the stool about 30 cm from a wall, and 2 m from the nearest corner in the room. There were no tables or other obstacles within a 4-m distance. The floor was carpeted and two large pieces ($4\text{ ft}\times 4\text{ ft}$) of absorbing foam material were attached to the wall behind the speakers to reduce reflections. Care was taken that reflections from the bodies of the data collectors did not interfere with the measured sound pressure levels.

When all four speakers were driven with the same polarity they acted as identical in-phase sources, and together as a monopole (omnidirectional) sound source. Figure 6(a) shows the measured directivity pattern for the monopole arrangement of the four speakers. The accuracy of the measured values is ± 0.5 dB. As expected, the monopole source radiates essentially the same in all directions.

When speaker pairs 1–2 and 3–4 had the opposite polarity the system acted as a dipole source (alternately, the speakers could have been paired as 1–4 and 2–3). Figure 6(b) shows the measured directivity pattern for the dipole arrangement of the four speakers. The solid curve represents the theoretical prediction for the dipole from Eq. (5) The agreement between theory and measurement is quite good.

When speaker pairs 1–3 and 2–4 had the opposite polarity the system acted as a lateral quadrupole source. Figure 6(c) shows the measured directivity pattern for the lateral quadrupole arrangement of the four speakers. The solid curve represents the theoretical prediction for the lateral quadrupole from Eq. (8). The agreement between theory and measurement is excellent.

To make a longitudinal quadrupole, the four speakers were equally spaced along a board on the stool. The outer pair and the inner pair of speakers had the opposite phase, as per Fig. 2(b). The sound level meter was moved back to a distance of

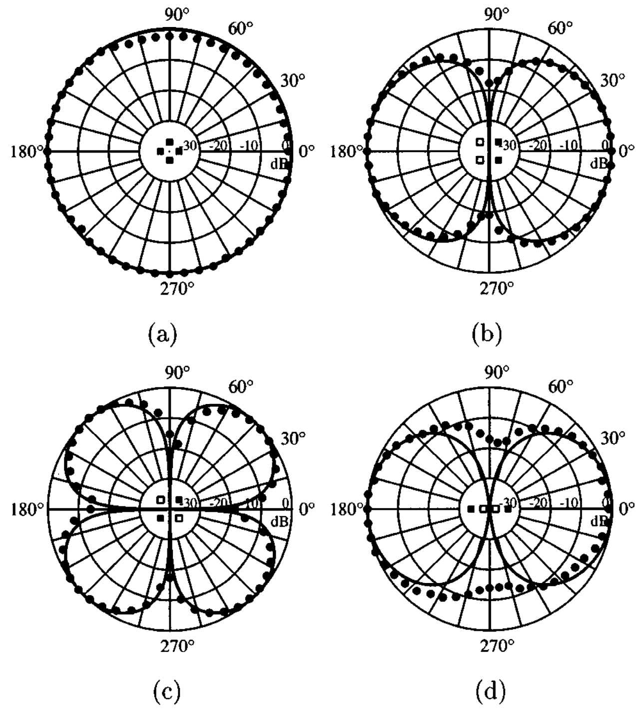

Fig. 6. Measured directivity patterns at 250 Hz for sound radiation from (a) monopole, (b) dipole, (c) lateral quadrupole, and (d) longitudinal quadrupole configurations of four 4-in. boxed loudspeakers. Speaker orientation and relative phase is indicated by the black and white boxes at the center of each plot.

2 m from the center of the speaker arrangement. Figure 6(d) shows the measured directivity pattern for the longitudinal quadrupole arrangement of the four speakers. The solid curve represents the theoretical prediction for the longitudinal quadrupole from Eq. (9). The agreement between theory and measurement is not as good for this arrangement. At this point, students are introduced to the concept of far field versus near field. A longitudinal quadrupole has a rather complicated near field and an observer must be in the deep far field $kr \gg 1$ for the approximation in Eq. (9) to be valid. Since our measurements were made at a distance of 2 m from a 250-Hz source, our value of $kr = 9.16$, which is not exactly much greater than 1. The directivity portion of the exact expression for the near field radiated by a longitudinal quadrupole source may be derived as[1,10]

$$p(r,\theta) \sim \left| (1 - 3\cos^2\theta)\left(\frac{ik}{r} - \frac{1}{r^2} + \frac{k^2}{3}\right) - \frac{k^2}{3} \right|, \qquad (10)$$

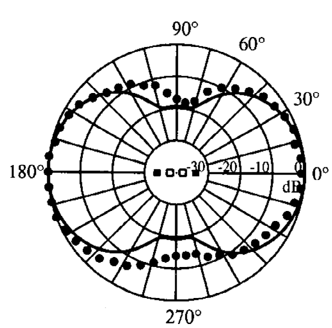

Fig. 7. Measured directivity patterns at 250 Hz for sound radiation from a longitudinal quadrupole compared with the exact near-field expression calculated for $r = 2$ m.

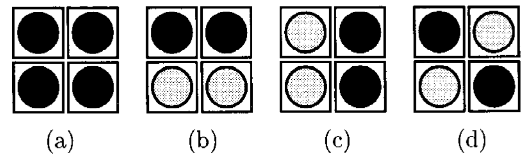

Fig. 8. Speaker arrangement and polarities for audible demonstration of sound power radiated by (a) monopole, (b) and (c) dipole, and (d) quadrupole sources.

which for large kr reduces to the directivity term in Eq. (9). Figure 7 shows the same measured data as in Fig. 6(d) compared with the exact near-field expression. Now the fit is much closer in the regions around 90° and 270°. Differences between measured values and theory are most likely due to reflections from equipment in the laboratory.

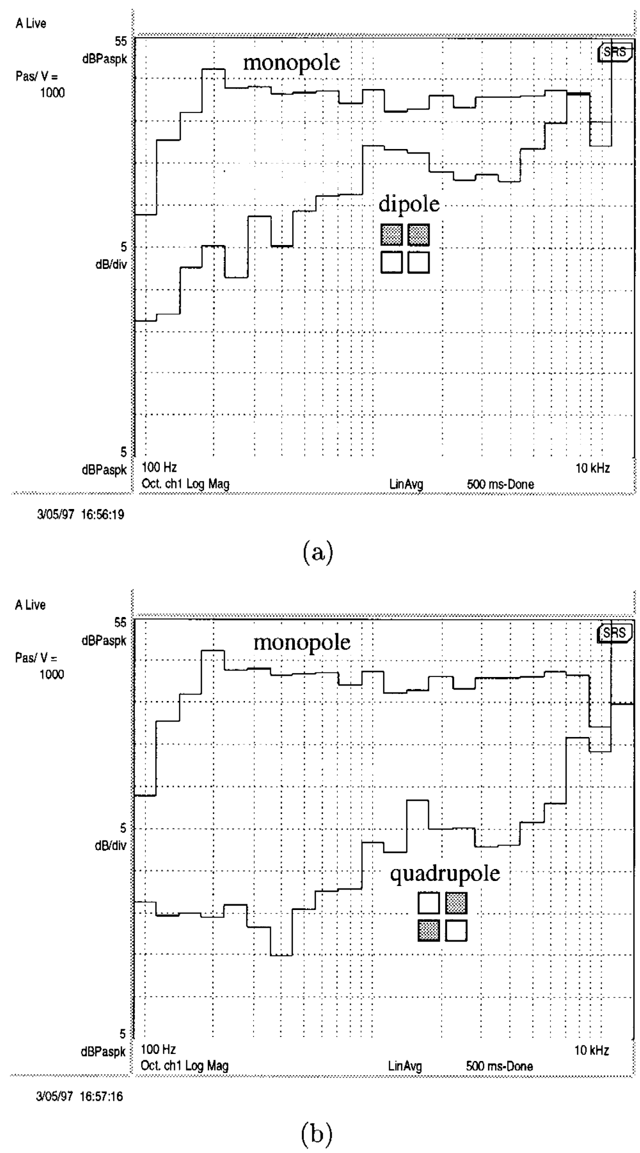

Fig. 9. Comparisons of 1/3-octave band measurement of pink noise played through four 4-in. speakers arranged as (a) monopole and dipole, (b) monopole and quadrupole.

IV. AUDIBLE DEMONSTRATION

A comparison of the effectiveness of monopole, dipole, and quadrupole sources at radiating low frequencies may be demonstrated very effectively using the apparatus described in this paper.[5,13] The speakers were stacked in a square all facing the same direction as shown in Fig. 8, and music was played through them (the same signal went to all four speakers). The polarity of individual speakers was reversed by the switch box. When the switches were set so that all four speakers had the same phase as in Fig. 8(a), so that the speaker arrangement acted like a monopole, the low frequencies in the music were quite audible. Reversing the polarity of two adjacent speakers produced a dipole source as in Fig. 8(b) and (c). This caused the bass frequencies to be significantly reduced while the middle and high frequencies were relatively unaffected. Matching the polarity of diagonal speakers produced a quadrupole source as in Fig. 8(d). Now the bass frequencies were severely impaired and midrange frequencies were also reduced. This very simple demonstration makes a big impression on students.

Figure 9 shows 1/3-octave measurements of the sound pressure level when pink noise was played through the speaker stack. A sound level meter was placed a distance of 1 m from the speakers and approximately aligned with the center of the square. Figure 9(a) compares the sound spectrum produced by monopole and dipole arrangements. The low frequencies are greatly reduced, with midfrequencies slightly reduced, and high frequencies largely unaffected. Figure 9(b) compares the sound spectrum produced by monopole and quadrupole arrangements. Now both low and middle frequencies have been severely reduced.

V. SUMMARY

A simple and inexpensive experiment using four 4-in. boxed loudspeakers very effectively demonstrates the angular distribution of the sound field radiated by acoustic monopole, dipole, and quadrupole sources. At low frequencies, each boxed speaker acts as a simple source, radiating sound equally in all directions. To observe directivity patterns the four speakers are symmetrically arranged on a rotating stool and the sound field is measured with a sound level meter as the stool is rotated through 360°. A simple switch box enables the polarity of each speaker to be reversed, allowing students to drive the speakers together as a monopole, or in pairs as dipoles and quadrupoles. Measured directivity patterns agree very well with theoretical predictions. Stacking the speakers and playing music through them demonstrates the relative efficiency of monopoles, dipoles, and quadrupoles at producing low frequency sounds.

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[10]D. D. Reynolds, *Engineering Principles of Acoustics: Noise and Vibration Control* (Allyn and Bacon, Boston, 1981), pp. 531–594.

[11]Optimus XTS 40 loudspeakers (4-in., 150–18 000 Hz) are available from Radio Shack (catalog No. 40-1991). They are quite often on sale for $14.99.

[12]Digital Sound Level Meter 33-2055 is available from Radio Shack for $59.99. An analog meter 33-2050 is available for $29.99.

[13]T. D. Rossing, *Acoustics Laboratory Experiments* (Northern Illinois U.P., DeKalb, IL, 1982), Experiment 41: Monopole, Dipole, and Quadrupole Sources.

PREMATURE DISCLOSURE

I wrote four letters explaining what I wanted and asking for hospitality. ...Back did not reply; Cohnen said that his grating was at the moment out of commission, because his institute was being rebuilt; Paschen told me that he liked my idea, and that he had just put one of his doctoral candidates to work on it. I was furious at this unexpected answer, but the project must have come to naught, because I never heard any further news of it. Zeeman, a Nobel Prize winner and the discoverer of the celebrated Zeeman effect, told me to catch a train and come to Holland.

Emilio Segrè, *A Mind Always in Motion—The Autobiography of Emilio Segrè* (University of California Press, Berkeley, 1993), pp. 65–66.