Backscattering from a baffled finite plate strip with fuzzy attachments

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A formulation for predicting the effect of fuzzy attachments on the response of a master structure was originally provided by the fuzzy structure theory of Soize. An extension of Soize's theory was recently developed by Pierce *et al.* (ASME 1993 Winter Meeting, New Orleans, LA). This new formulation is applied to a finite plate strip simply supported in an infinite rigid baffle. An incident plane-wave pulse is incident upon the plate and the effect of the fuzzy attachments on the target strength is determined. The primary effect of a large number of 1-DOF fuzzy attachments is an apparent added mass and an apparent added damping to the plate. Both of these effects depend directly on the mass distribution of the 1-DOF attachments with respect to their natural frequencies. A representative distribution is considered. It is found that if the most probable natural frequency of the fuzzy attachments coincides with a plate resonance, the amplitude at the target frequency is significantly reduced, the amount of reduction increasing as the total mass of all attachments increases. Plate resonances above the target frequency are shifted upward, and those below are shifted downward. © *1995 Acoustical Society of America.*

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INTRODUCTION

The vibration reduction, or damping, effect of adding a single one-degree-of-freedom (1-DOF) oscillator to a vibrating structure is well documented in the literature. Of more recent interest is the effect of attaching a large number of 1-DOF oscillators to a vibrating structure. Kobelev[1] investigated the effect of a large number of 1-DOF oscillators attached to a massless wall upon which a pressure wave is normally incident. The damping constants of the attachment were assumed to be negligible, and the stiffness and modal density of the attachments were known. The main effect of the attachments is an effective damping, analogous to Landau damping in plasma physics. Xu and Igusa[2] investigated a 1-DOF mass-spring system to which a large number of 1-DOF substructures are attached. In this formulation the attachments have known masses and damping constants, and the natural frequencies are evenly and closely spaced over a frequency band surrounding the resonance frequency of the main oscillator. The primary effect observed was that the attachments provided an effective damping to the main oscillator. While these results are very important, both analyses assume a rather complete knowledge of the attachment parameters.

Soize[3–5] introduced the concept of a *fuzzy structure*, separating the complex structure under investigation into two parts: the master structure, which is known and can be modeled conventionally, and the fuzzy substructure which is imprecisely known and cannot be modeled conventionally. The effects of the fuzzy substructure are described by an equivalent boundary impedance, the real and imaginary parts of which correspond to an added damping and mass loading, respectively. The fuzzy substructure is modeled as a system of 1-DOF oscillators whose mass M_n, damping ξ_n, and natural frequencies Ω_n are known only in a probabilistic sense (i.e., mean values with random dispersion). The ensuing mathematical formulation is designed for finite element analysis, and requires a midfrequency signal processing technique.[6] Soize's fuzzy structure theory provides response spectra that compare well with simulations of actual structures with complicated internal attachments. Application of Soize's fuzzy structure theory to canonical problems is somewhat hindered, however, by the difficulty of the mathematics involved and by the signal processing method.

Recent extensions to Soize's theory have resulted in the study of simpler structures with fuzzy attachments. Feit and Strasberg[7] have developed a somewhat less fuzzy method along lines similar to the formulation of Kobelev. The present authors were privileged to collaborate with Pierce[8] on a simplification of Soize's fuzzy structure theory which yields promising results and is straightforward to implement. In this paper we will briefly review the important details and results of this simpler fuzzy model, and then apply these results to an investigation of the backscatter from a baffled plate strip to which a large number of fuzzy substructures are attached.

I. A BRIEF REVIEW OF PIERCE'S FUZZY FORMULATION

The model of the attachments used in the present paper is based entirely on the extension of Soize's fuzzy structure theory developed by Pierce[8] and the present authors. The main difference between this new description of a fuzzy attachment and the original theory of Soize is the manner in which the fuzzy attachments are modeled. As was the case in the theory of Soize, the fuzzy attachments are modeled as 1-DOF oscillators. The parameters of these oscillators are

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governed by a *principle of maximum ignorance*. That is, one does not know the total number of attachments N, though N is assumed to be large, nor does one know their exact locations (x_n, y_n), or the mass M_n, damping ζ_n, and natural frequencies Ω_n of the individual attachments. What is known, however, is the distribution of the masses of the attachments with respect to their natural frequencies. This information is contained in the function $\bar{m}_F(\Omega)$ which represents the combined mass (per unit area) of all fuzzy attachments which have natural frequencies below some specified frequency Ω. The quantity $\bar{m}_F(\infty)$ would then represent the total mass of all attachments. Perhaps even more important than $\bar{m}_F$ is the derivative of $\bar{m}_F$ with respect to Ω, $d\bar{m}_F/d\Omega$. This mass-frequency density is used in lieu of modal density which plays a key role in statistical energy analysis, in the problem studied by Kobelev,[1] and in the fuzzy structure theory of Soize.[3–5]

The present model of the fuzzy attachments removes the need for detailed knowledge of their dynamical response. Instead, one only needs to know the forces they exert on the master structure. For the problem investigated in the present paper, a transient sound pulse is incident upon a baffled plate strip, to the underside of which are attached N fuzzy attachments. The governing equation for this system is then

$$-\omega^2 m_{\text{pl}} w + B_{\text{pl}} \frac{d^4 w}{dx^4} = -p(x,y,0,\omega) - \sum_{n=1}^{N} F_n \delta(x - x_n), \tag{1}$$

where the left-hand side represents the bending motion of the plate, the first term on the right is the pressure on the surface of the plate, and the second term on the right accounts for the effect of the fuzzy attachments.

The Pierce *et al.* extension of Soize's fuzzy structure theory employs a spatial averaging over patches small compared to a relevant wavelength, and a frequency averaging over narrow frequency bands. Also, it assumes transient excitation in the time domain, and transforms the response to the frequency domain.

The primary result is that the fuzzy attachments add an apparent mass and an apparent damping to the master structure. If it is assumed, for simplicity, that the damping of all the attachments is the same, ζ, then the effect of the fuzzy attachments appear to the master structure as an apparent added mass per unit area

$$m_{F,\text{appar}} = \int_0^\infty \frac{d\bar{m}_F}{d\Omega} \left\{ \frac{\Omega^2[\Omega^2 - \omega^2 + 4\omega^2\zeta^2]}{[\Omega^2 - \omega^2]^2 + [2\zeta\omega\Omega]^2} \right\} d\Omega, \tag{2}$$

and an apparent added damping (units of force per unit area divided by velocity)

$$R_{F,\text{appar}} = \int_0^\infty \frac{d\bar{m}_F}{d\Omega} \left\{ \frac{2\zeta\omega^4\Omega}{[\Omega^2 - \omega^2]^2 + [2\zeta\omega\Omega]^2} \right\} d\Omega, \tag{3}$$

where ω is a frequency component of the transient pulse driving the master structure.

If the damping of the fuzzy attachments is taken to be very small, then Pierce[8] shows that the asymptotic limits of $m_{F,\text{appar}}$ and $R_{F,\text{appar}}$ as $\zeta \to 0$ become

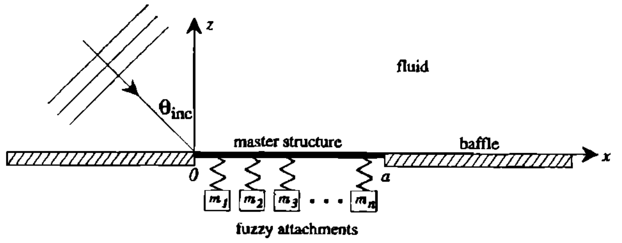

FIG. 1. Plate strip with internal fuzzy attachments, simply supported in an infinite baffle.

$$m_{F,\text{appar}} = \text{Pr} \int_0^\infty \frac{d\bar{m}_F}{d\Omega} \frac{\Omega^2}{\Omega^2 - \omega^2} d\Omega, \tag{4}$$

$$R_{F,\text{appar}} = \frac{\pi\omega^2}{2} \frac{d\bar{m}_F}{d\Omega} \bigg|_{\Omega=\omega}, \tag{5}$$

where Pr indicates that the principal value of the integral is to be taken. The resulting expression for the apparent added damping in Eq. (5) has also been obtained by Feit and Strasberg,[7] and is similar to the results of Kobelev.[1] The fuzzy structure theory of Soize[3–5] also predicts an added damping term, though one must look rather hard to find it in the form similar to Eq. (5) [see, for example, the expression for $\beta_o^l(\omega)$ on p. 855 of Ref. 5].

So then, the effect of the fuzzy attachments may be accounted for simply by adding to the mass of the master structure a frequency-dependent mass term and by adding a frequency-dependent damping to the dynamics of the master structure.

II. THEORETICAL FORMULATION

The problem being investigated in the present paper involves a plate of width a and of infinite length, simply supported in an infinite rigid baffle. The plate is taken as an Euler–Bernoulli–Kirchhoff plate with mass per unit area m_{pl} and bending modulus B_{pl}. On the underside of the plate are attached N fuzzy structures. Fluid exists only in the $z>0$ half-space, and is assumed to be light (air). A plane-wave pulse is incident on the plate and it is desired to obtain the far-field echo received in the backscattered direction. The goal is to observe the effect of the fuzzy attachments on the target strength. A cross section in the x-z plane of this scenario is shown in Fig. 1.

A. Plate without attachments

First consider the response of the plate alone, without attachments. The frequency domain equation governing the dynamics of the plate strip responding to an incident wave pulse is

$$-\omega^2 m_{\text{pl}} \hat{w} + B_{\text{pl}} \frac{d^4 \hat{w}}{dx^4} = -\hat{p}(x,y,0,\omega), \tag{6}$$

where $\hat{w}$ is the Fourier transform (time⇔frequency) of the normal plate displacement w. The term on the right-hand side of (6) represents the total pressure at the surface of the plate strip. The fluid in the half-space $z>0$ has ambient density ρ and sound speed c. The vibrations of the structure and perhaps some external disturbance give rise to a fluctuating pressure field in this half-space, which is described by a pressure field $\hat{p}(x,y,z,\omega)$. If the plate were rigid, then the surface would be a perfectly reflecting rigid plane, and the blocked pressure field which results in this limiting case is denoted by $\hat{p}_{\text{bl}}(x,y,z,t)$. Because this external field causes the plate to vibrate, there is an additional radiated contribution to the overall pressure disturbance, so that

$$\hat{p}(x,y,z,\omega)=\hat{p}_{\text{bl}}(x,y,z,\omega)+\hat{p}_{\text{rad}}(x,y,z,\omega). \tag{7}$$

This radiated wave can be regarded as caused by the plate's vibrations, in the sense that if one knew the z component of the plate displacement $\hat{w}(x,y,\omega)$ explicitly, then one could calculate the radiated wave by an appropriate version of the Rayleigh integral[9]

$$\hat{p}_{\text{rad}}(x,y,z,\omega)=-\frac{\omega^2\rho}{2\pi}\int\int \hat{w}(\xi,\eta,\omega)\,\frac{e^{ikR}}{R}\,d\xi\,d\eta, \tag{8}$$

where

$$R=[(x-\xi)^2+(y-\eta)^2+z^2]^{1/2} \tag{9}$$

is the distance from the integration point on the plate to the listener point. If the region of analysis is taken to be the $y=0$ plane, then (8) becomes

$$\hat{p}_{\text{rad}}(x,z,\omega)=-\frac{\omega^2\rho}{2\pi}\int_0^a \hat{w}(\xi,\omega)\int_{-\infty}^{\infty}\frac{e^{ik(R^2+\eta^2)^{1/2}}}{(R^2+\eta^2)^{1/2}}\,d\eta\,d\xi, \tag{10}$$

where $R=[(x-\xi)^2+z^2]^{1/2}$ and

$$\int_{-\infty}^{\infty}\frac{e^{ik(R^2+\eta^2)^{1/2}}}{(R^2+\eta^2)^{1/2}}\,d\eta=i\pi H_0^{(1)}(kR) \tag{11}$$

with $H_0^{(1)}$ being the zeroth-order Hankel function of the first kind.[10] Thus the pressure radiated by the strip becomes

$$\hat{p}_{\text{rad}}(x,z,\omega)=-\frac{i\omega^2\rho}{2}\int_0^a \hat{w}(\xi,\omega)H_0^{(1)}(kR)d\xi \tag{12}$$

and, evaluated at the surface of the strip,

$$\hat{p}_{\text{rad}}(x,0,\omega)=-\frac{i\omega^2\rho}{2}\int_0^a \hat{w}(\xi,\omega)H_0^{(1)}(k|x-\xi|)d\xi. \tag{13}$$

If the incident wave has the form

$$\hat{p}_{\text{inc}}(x,z,\omega)=\hat{P}_o e^{i(\omega/c)x\sin\theta_{\text{inc}}}e^{-i(\omega/c)z\cos\theta_{\text{inc}}}, \tag{14}$$

then the blocked pressure term is

$$\hat{p}_{\text{bl}}(x,0,\omega)=2\hat{P}_o e^{i[\omega x/c]\sin\theta_{\text{inc}}} \tag{15}$$

and the total pressure at the surface of the plate strip is

$$\hat{p}(x,y,0,\omega)=2\hat{P}_o e^{i[\omega x/c]\sin\theta_{\text{inc}}} -\frac{i\omega^2\rho}{2}\int_0^a \hat{w}(\xi,\omega)H_0^{(1)}(k|x-\xi|)d\xi. \tag{16}$$

In the far field, taking r as the distance from the observation point to the origin,

$$H_0^{(1)}(kR)\rightarrow\left(\frac{2c}{\pi\omega}\right)^{1/2}\frac{1}{\sqrt{r}}\,e^{-i(\pi/4)}e^{ikr}e^{-i(k/r)x\xi}. \tag{17}$$

If the backscatter direction is taken to be $x/r=-\sin\theta_{\text{inc}}$, then the far-field radiated pressure becomes

$$\hat{p}_{\text{rad}}(\omega)=\left(\frac{\rho^2c\omega^3}{2\pi}\right)^{1/2}e^{-i(\pi/4)}\frac{e^{i(\omega/c)r}}{\sqrt{r}} \times\int_0^a \hat{w}(\xi,\omega)e^{i(\omega/c)\xi\sin\theta_{\text{inc}}}\,d\xi. \tag{18}$$

One can then define the backscattering cross section in two dimensions as

$$\sigma_{\text{back}}=\lim_{r\rightarrow\infty}2\pi r\,\frac{|\hat{p}_{\text{rad}}(\omega)|^2}{\hat{P}_o^2}. \tag{19}$$

The target strength is appropriately defined for the two-dimensional panel as

$$\text{TS}=10\log\frac{\sigma_{\text{back}}}{2\pi R_{\text{ref}}}, \tag{20}$$

where R_{ref} is taken to be 1 m.

In order to calculate the backscattering cross section and target strength, the normal displacement of the plate must be specified. The simply supported boundary conditions for the plate at $x=0$ and $x=a$ require that w and $\partial^2w/\partial x^2$ vanish. This suggests that the spatial dependence of the normal displacement will be sinusoidal. Since the spatial dependence is not affected by the time/frequency Fourier transform we can assume that the normal displacement is of the form

$$\hat{w}(x)=\sum_{n=1}^{\infty}A_n\sin\left(\frac{n\pi}{a}x\right), \tag{21}$$

where the A_n are unknown.

Substituting this form of the normal displacement into (19) yields the backscattering cross section

$$\sigma_{\text{back}}=\omega^3\rho^2c\left|\sum_n\frac{A_n}{\hat{P}_o}\int_0^a\sin\left(\frac{n\pi}{a}\xi\right)e^{ik\xi\sin\theta_{\text{inc}}}\,d\xi\right|^2. \tag{22}$$

The ratio $A_n/\hat{P}_o$ may be evaluated for the strip, without attachments, from an analysis of the governing equation (6). Substitution of Eq. (21), and applying the principal of superposition, one can see that for the nth spatial mode,

$$\left[-\omega^2m_{\text{pl}}+B_{\text{pl}}\left(\frac{n\pi}{a}\right)^4\right]A_n\sin\left(\frac{n\pi}{a}x\right) =-2\hat{P}_o(\omega)e^{ikx\sin\theta_{\text{inc}}}+\frac{i\omega^2\rho}{2}\int_0^a A_n\sin\left(\frac{n\pi}{a}\xi\right) \times H_0^{(1)}(k|x-\xi|)d\xi. \tag{23}$$

It must be noted that this approach considers the fluid loading to be light. That is, the modes of the plate are not coupled to each other through the fluid, but rather each mode may be analyzed independently of all other plate modes. For heavy fluid loading the summation in (21) would have to be included in (23), thus coupling each mode to every other mode, and the problem becomes substantially more difficult.

Multiplying (23) through by $\sin[(p\pi/a)x]$ and then integrating over x and making use of the orthogonality of the spatial distribution function yields

$$\left[-\omega^2 m_{\text{pl}}+B_{\text{pl}}\left(\frac{n\pi}{a}\right)^4\right]A_n\left(\frac{a}{2}\right)-\frac{i\omega^2\rho}{2}A_n\int_0^a\int_0^a\sin\left(\frac{n\pi}{a}\xi\right)\times\sin\left(\frac{n\pi}{a}x\right)H_0^{(1)}(k|x-\xi|)d\xi\,dx=-2\hat{P}_o(\omega)\int_0^a\sin\left(\frac{n\pi}{a}x\right)e^{ikx\sin\theta_{\text{inc}}}dx. \tag{24}$$

The only unknowns are A_n and $\hat{P}_o$; solving for the ratio $A_n/\hat{P}_o$ yields

$$\frac{A_n}{\hat{P}_o}=-2\int_0^a\sin\left(\frac{n\pi}{a}x\right)e^{ikx\sin\theta_{\text{inc}}}dx\left\{\left[-\omega^2 m_{\text{pl}}+B_{\text{pl}}\left(\frac{n\pi}{a}\right)^4\right]\frac{a}{2}-\frac{i\omega^2\rho}{2}\int_0^a\int_0^a\sin\left(\frac{n\pi}{a}\xi\right)\times\sin\left(\frac{n\pi}{a}x\right)H_0^{(1)}(k|x-\xi|)d\xi\,dx\right\}^{-1}. \tag{25}$$

This ratio is the key to the backscattering cross section and thus the target strength. By defining two constants

$$C_1=\frac{B_{\text{pl}}\pi^4}{a^2c^2m_{\text{pl}}}\quad\text{and}\quad C_2=\frac{\rho a}{m_{\text{pl}}}, \tag{26}$$

the denominator may be rearranged so that it does not explicitly depend on the plate parameters,

$$\frac{A_n}{\hat{P}_o}=-\left(\frac{4}{a\omega^2m_{\text{pl}}}\right)\int_0^a\sin\left(\frac{n\pi}{a}x\right)e^{ikx\sin\theta_{\text{inc}}}dx\times\left\{\left[\frac{n^4C_1}{(ka)^2}-1\right]-i\frac{C_2}{a^2}\int_0^a\int_0^a\sin\left(\frac{n\pi}{a}\xi\right)\times\sin\left(\frac{n\pi}{a}x\right)H_0^{(1)}(k|x-\xi|)d\xi\,dx\right\}^{-1}. \tag{27}$$

C_1 controls the location of the plate resonance frequencies and C_2 controls the effect of the radiation damping (fluid loading). For a 1-cm-thick, 1-m-wide steel plate strip in air, $C_1\approx0.1$ and $C_2\approx0.01$.

B. Plate with fuzzy attachments

Now consider the plate with N fuzzy attachments on the underside. The fuzzy attachments add an apparent mass and an apparent damping to the plate, defined in (4) and (5). The governing equation (6) then becomes

$$-\omega^2[m_{\text{pl}}+m_{F,\text{appar}}]\hat{w}-i\omega R_{F,\text{appar}}\hat{w}+B_{\text{pl}}\frac{d^4\hat{w}}{dx^4}=-2\hat{P}(\omega)e^{i[\omega x/c]\sin\theta_{\text{inc}}}+\frac{i\omega^2\rho}{2}\int_0^a\hat{w}(\xi,\omega)\times H_0^{(1)}(k|x-\xi|)d\xi. \tag{28}$$

The equivalent of Eq. (27) for the plate with attachments is

$$\frac{A_n}{\hat{P}_o}=-\left(\frac{4}{a\omega^2m_{\text{pl}}}\right)\int_0^a\sin\left(\frac{n\pi}{a}x\right)e^{ikx\sin\theta_{\text{inc}}}dx\times\left\{\left[\frac{n^4C_1}{(ka)^2}-1-\frac{m_{F,\text{appar}}}{m_{\text{pl}}}-i\frac{R_{F,\text{appar}}}{\omega m_{\text{pl}}}\right]-i\frac{C_2}{a^2}\int_0^a\int_0^a\sin\left(\frac{n\pi}{a}\xi\right)\sin\left(\frac{n\pi}{a}x\right)\times H_0^{(1)}(k|x-\xi|)d\xi\,dx\right\}^{-1}. \tag{29}$$

The backscattering cross section and target strength may then be calculated using (19) and (20).

III. PROTOTYPE MASS-FREQUENCY DISTRIBUTION

In Ref. 8 we consider a prototype mass-frequency distribution in order to study the physical concepts governing the effects of the apparent mass and damping due to the fuzzy attachments. This mass-frequency distribution has the form

$$\bar{m}_F(\Omega)=\bar{m}_F(\infty)[1-e^{-\Omega^2/2\Omega_F^2}], \tag{30}$$

so that the mass-frequency density function is

$$\frac{d\bar{m}_F}{d\Omega}=\bar{m}_F(\infty)\frac{\Omega}{\Omega_F^2}e^{-\Omega^2/2\Omega_F^2}. \tag{31}$$

Here $\bar{m}_F(\infty)$ represents the total mass of all attachments, and Ω_F is the most probable natural frequency of the fuzzy attachments. Figure 2 shows the suggested mass-frequency distribution and density, in nondimensional form.

With this mass-frequency distribution, the apparent frequency-dependent mass per unit area added by the fuzzy attachments (4), in the limit $\zeta\rightarrow0$, is

$$m_{F,\text{appar}}=\bar{m}_F(\infty)\Pr\int_0^\infty e^{-\Omega^2/2\Omega_F^2}\frac{\Omega^2}{\Omega^2-\omega^2}\left(\frac{\Omega}{2\Omega_F^2}\right)d\Omega \tag{32}$$

and the apparent added damping (5) becomes

$$R_{F,\text{appar}}=\frac{\pi\omega^2}{2}\bar{m}_F(\infty)\frac{\omega}{\Omega_F^2}e^{-\omega^2/2\Omega_F^2}. \tag{33}$$

Using the substitutions

$$u=\frac{\Omega^2}{2\Omega_F^2}\quad\text{and}\quad\eta=\frac{\omega^2}{2\Omega_F^2}, \tag{34}$$

the apparent added damping becomes

$$R_{F,\text{appar}}=\pi\omega\bar{m}_F(\infty)\eta e^{-\eta} \tag{35}$$

and the apparent added mass simplifies to

$$m_{F,\text{appar}}=\bar{m}_F(\infty)\Pr\int_0^\infty\frac{u}{u-\eta}e^{-u}\,du=\bar{m}_F(\infty)[1-\eta e^{-\eta}\,\text{Ei}(\eta)], \tag{36}$$

where

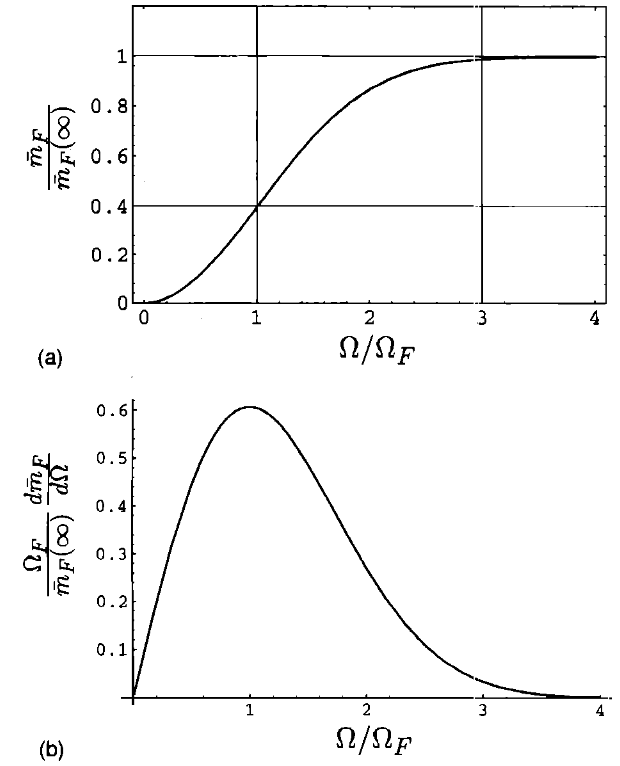

FIG. 2. (a) Prototype mass-frequency distribution for the fuzzy attachments. Ω represents the natural frequency of an individual attachment, Ω_F represents the most probable natural frequency of the fuzzy attachments, and $\bar{m}_F(\infty)$ is the total mass of all attachments. (b) Mass-frequency density $d\bar{m}_F/d\Omega$.

$$\mathrm{Ei}(\eta)=-\mathrm{Pr}\int_{-\eta}^{\infty}\frac{e^{-\xi}}{\xi}\,d\xi \tag{37}$$

is the exponential integral function.[11]

Figure 3 shows the apparent mass per unit area added to the plate by the fuzzy attachments, normalized to the total mass of all attachments and plotted versus the frequency (normalized to the most probable attachment natural frequency Ω_F) at which the plate is vibrating. In the limit of zero frequency the apparent added mass is equal to the total mass of all attachments; the attachments appear to be rigidly connected to the plate. As the frequency increases above zero the apparent added mass is greater than the total mass of the attachments. In this frequency regime the majority of the attachments are being driven below their natural frequencies so that they are moving in phase with the plate, causing their displacements to be larger than that of their attachments points and thus their kinetic energies to be larger than if they were rigidly attached. Once the driving frequency increases beyond $\omega=0.863\Omega_F$ the apparent added mass drops below the total attached mass. The apparent added mass becomes negative when $\omega=1.641\Omega_F$ and remains negative for all higher frequencies. Now most of the attachments are being driven at frequencies higher than their natural frequencies and their corresponding motion is in opposite phase to that of the plate. The forces exerted on the plate by the attachments are in the same direction as the plate acceleration, as if mass were being subtracted from the plate. An alternate interpretation of this result is that the fuzzy attachments provide an added stiffness to the master structure.

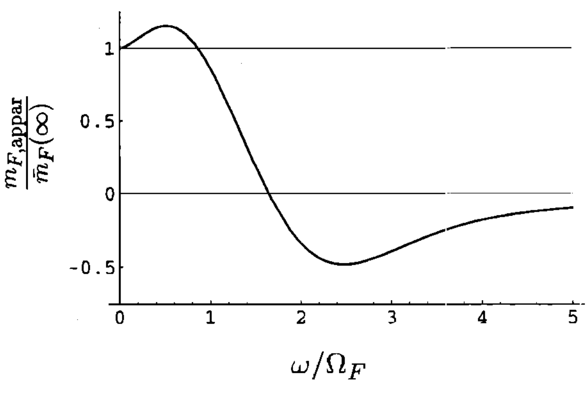

FIG. 3. Normalized apparent mass per unit area added to the plate strip by the fuzzy attachments, in the limit $\zeta\rightarrow 0$, when the plate is vibrating with frequency ω.

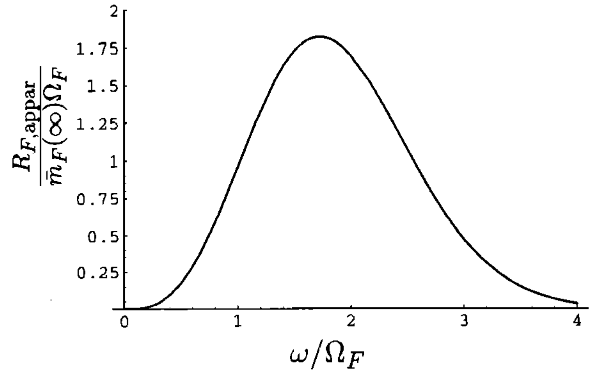

FIG. 4. Normalized apparent damping constant added to the plate strip by the fuzzy attachments, in the limit $\zeta\rightarrow 0$, when the plate is vibrating with frequency ω.

Figure 4 shows the normalized apparent damping added to the plate by the fuzzy attachments, plotted versus the normalized frequency at which the plate is being driven. While the prototype mass-frequency distribution function (30) may seem somewhat *ad hoc*, the apparent added mass and damping as shown in Figs. 3 and 4 are representative of what one might expect for a substructure consisting of a large number of 1-DOF oscillators.

IV. COMPUTATIONAL FORMULATION

All numerical computations in this paper were performed using MATHEMATICA[12] on a NeXTstation Turbo (68040 @ 33 MHz). The integral

$$\int_0^a\int_0^a \sin\left(\frac{n\pi}{a}\xi\right)\sin\left(\frac{n\pi}{a}x\right)H_0^{(1)}(k|x-\xi|)d\xi\,dx \tag{38}$$

in the denominator of Eqs. (27) and (29) proved to be a very time-consuming calculation using NINTEGRATE, MATHEMATICA's numerical integration function.[13] A typical target strength plot like those shown in Fig. 5 took over 11 h to generate. To speed up the process, the integral in (38) was evaluated at ten points over the frequency range of interest, and the results for these ten points were fit with a polynomial. This polynomial fit was then used in place of the integral for determining the target strength, with the awareness

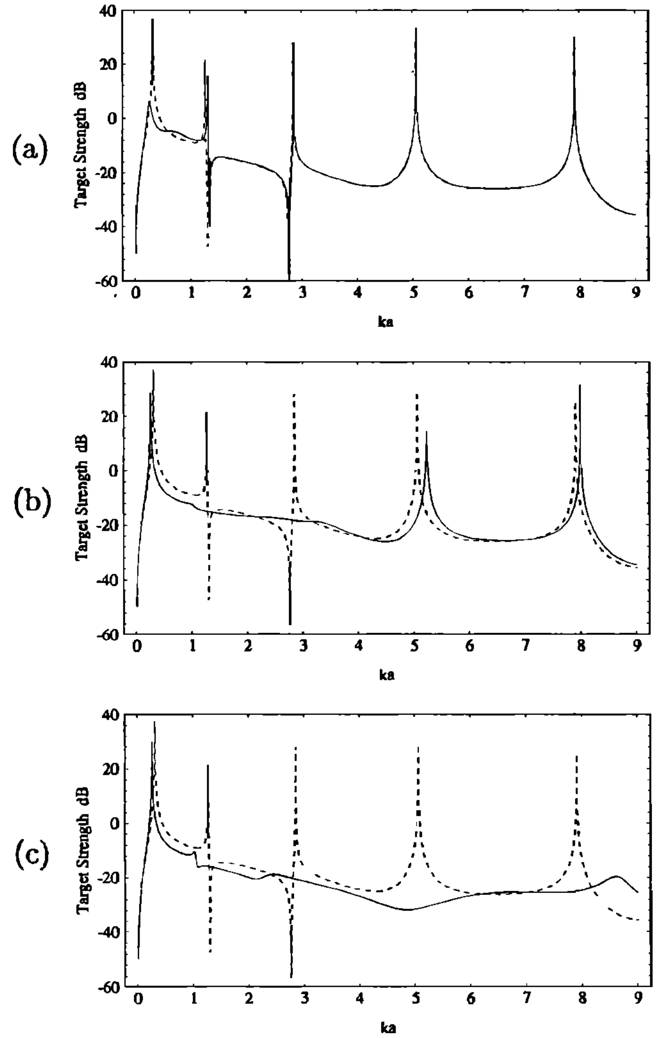

FIG. 5. The dashed curves represent the target strength at 45° for the plate without any attachments. The solid curves represent the target strength at 45° for the plate with fuzzy attachments. The total mass of the fuzzy attachments is 40% of the plate mass. (a) $K_Fa=0.316$, (b) $K_Fa=1.265$, (c) $K_Fa=2.846$.

that such a fit is valid only for the range of frequencies over which the integral was evaluated. Using the polynomial fit reduced the computation time to generate a single target strength plot from over 11 h to under 5 min, with no discernible difference in plot accuracy.

V. RESULTS

The plots of target strength versus ka in this section were calculated for a steel plate strip in air. The plate is 1 m wide and 1 cm thick. Figure 5 shows the effect of the fuzzy attachments on the target strength of the plate for a 45° incident plane wave. The dashed curves represent the target strength for the plate without any attachments and the solid curves represent the target strength for the plate with fuzzy attachments. The total mass of the fuzzy attachments is 40% of the plate mass. The most probable natural frequency of the attachments, Ω_F, has been nondimensionalized as $\Omega_F a/c = K_F a$. In Fig. 5(a) $K_Fa=0.316$, so that the most probable natural frequency of the attachments coincides with the first resonance frequency of the plate. As shown in the

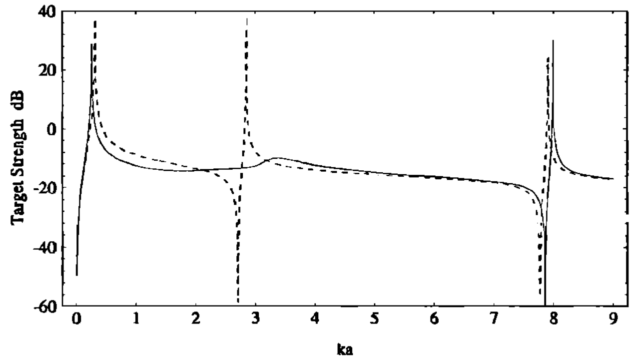

FIG. 6. A comparison between the backscatter from the plate with (solid curve) and without (dashed curve) attachments for normal incidence. The total mass of the attachments is 40% of the plate mass, and $K_Fa=1.265$.

plot, the first plate resonance is reduced in amplitude and lowered in frequency, while the other plate resonances are mostly unaffected. In Fig. 5(b) $K_Fa=1.265$, so that the most probable natural frequency of the attachments coincides with the second resonance frequency of the plate. Now the first plate resonance is slightly lower in frequency, the fourth and fifth plate resonances are shifted higher in frequency, and the second and third resonances are completely damped out. In Fig. 5(c) $K_Fa=2.846$, so that the most probable natural frequency of the attachments coincides with the third resonance frequency of the plate. The first plate resonance is again slightly lower in frequency, the second resonance is greatly reduced in amplitude and lower in frequency, the fifth peak is shifted higher in frequency with greatly reduced amplitude, and the third and fourth peaks have been removed. All three cases agree with what one would expect in light of the apparent added mass and damping of the form shown in Figs. 3 and 4. For frequencies below Ω_F ($ka<K_Fa$) the apparent total combined mass of plate and attachments is larger than the actual combined mass, thus causing resonance frequencies to shift downward in frequency. Similarly, at frequencies above Ω_F ($ka>K_Fa$) the apparent total combined mass of plate and attachments is less than the actual combined mass, thus causing resonance frequencies to shift upward in frequency. For frequencies within the range $1<\omega/\Omega_F<3$ ($K_Fa<ka<3K_Fa$) the apparent added damping is significant, severely damping any resonance peaks within this range.

Figure 6 compares the backscatter from the plate without attachments (dashed curve) and with attachments (solid curve) for normal incidence. The total mass of the attachments is 40% of the plate mass and $K_Fa=1.265$. As expected for normal incidence, the even numbered modes of the simply supported strip are not excited, and thus they are absent in both curves.

Figure 7 compares the effect of light and heavy fuzzy attachments. The dashed curve shows the target strength from the plate when the total mass of the attachments is 1% of the plate mass. The solid curve is for a total attachments mass equal to the plate mass. For both curves the angle of incidence is 45° and $K_Fa=1.265$. As the total attached mass increases, the peaks above and below K_Fa are shifted upward and downward, respectively, with frequency, and the

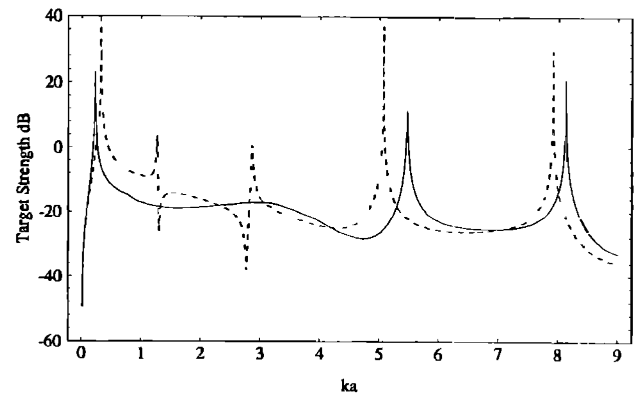

FIG. 7. Dependence of target strength on total mass of attachments: (dashed) total mass of attachments is 1% of plate mass; (solid) total attached mass equals plate mass. $K_F a = 1.265$.

peaks in the neighborhood of $K_F a$ are damped out completely. Increasing the total mass of the fuzzy attachments amplifies their effect (i.e., greater damping of targeted modes and greater frequency shift of resonance frequencies). However, it is the distribution of the attachments' mass with natural frequency that determines which modes are damped out and which are shifted in frequency. This suggests that the distribution of the mass of the attachments with respect to their natural frequencies plays a more important role than the total mass of the attachments.

VI. CONCLUSIONS

The extension of Soize's fuzzy structure theory, developed by Pierce *et al.*,[8] predicts that the primary effects of a large number of 1-DOF fuzzy attachments are an added frequency-dependent mass per unit area and an added frequency-dependent damping to the structure to which they are attached. Application to canonical problems is very easy; simply add these terms to the governing equation of the system under investigation. In this paper the investigated system was a finite width infinite length plate strip simply supported in an infinite, rigid baffle, with attachments underneath, and interrogated by an incident plane pulse. The effect of the fuzzy attachments was determined through analysis of the backscattered target strength.

Analyzing the mass loading effect of the fuzzy attachments, it was seen that for fuzzy attachments being driven below their natural frequencies the apparent added mass is positive, thus lowering the resonance frequencies of the first few modes of the plate. For fuzzy attachments being driven above their natural frequencies this apparent added mass is negative, raising the resonance frequencies of the higher modes of the plate. The fuzzy attachments also prov de an apparent added damping to the plate. The most probable natural frequency of the attachments may be adjusted so that this added damping completely wipes out selected plate resonances.

It should be emphasized that the long-term goal of this research is not to determine the effects of backscattering from objects with attached 1-DOF oscillators. In many vibroacoustic problems complicated substructures are often inaccessible to conventional modeling. The question then becomes how one simply account for these attached substructures and their effect on the primary (master) structure. In this paper it has been shown that the Pierce *et al.* extension to Soize's fuzzy structure theory does, in fact, result in an apparent damping which can significantly alter backscattering from a complex structure. Having passed the first test, therefore, it is clear that additional testing and refinement of the Pierce *et al.* extension to Soize's theory should be undertaken.

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